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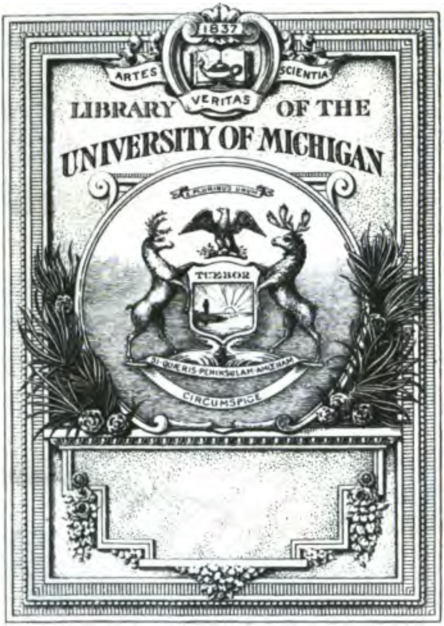
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Clavis Uſuræ;

Or, A KEY to

INTEREST

BOTH

Simple and Compound:

CONTAINING

Practical *Rules*, plainly expreſs'd in *Words at length* whereby, all the various *Caſes* of *Intereſt*, and *Annuities*, or *Leaſes* either in Poſſeſſion, or Reverſion and Purchaſing *Free-hold Eſtates*, &c. may very eaſily be Reſolv'd, both by the *Pen* and a ſmall *Table* of *Logarithms*, hereunto annex'd; For all *Rates* of *Intereſt*, and *Times* of *Payment* whatſoever; Illuſtrated by Variety of *Examples*.

To which is Added,

Rules to be Obſerv'd in Eſtimating the *Value* of *Annuities*, or *Leaſes*, and *Inſurances* for *Lives*, &c.

Alſo,

The Buſineſs of *Rebate* or *Diſcompt*, and the *Equation* of *Payments* (very uſeful for Merchants and other Dealers) is here *Rectified* and truly *Determin'd*.

Dealers

By J.ohn WARD.

LONDON:

Printed by *J. M.* for *William Taylor*, at the *Ship*
in *Pater-Noſter-Row*, near the North Door of St.
Paul's Church. 1710.

(Price 2 *s.*)

Hist. of Sci.
Huntting
4-10-76
12964

To the Honourable
Sir *John Wentworth*,
OF
NORTH-ELMS-HALL,
IN THE
Weſt-Riding of *Yorkſhire*,
BARONET;

This TRACT, as an Acknowledgment of Great Favours and Obligations receiv'd,

Is moſt humbly Dedicated,

and Preſented,

By

J. W.

THE

PREFACE.

IT *may be here expected that I ſhould (according to the Cuſtom of* Prefaces) *give the Reader ſome Account of the inſuing Pages, and of the Motives which induced me to Publiſh them; more eſpecially, becauſe there is already ſo many Books which Treat upon the ſame Subject.*

'Tis true indeed, the Buſineſs of Intereſt *and* Annuities, &c. *has been handled by ſeveral Authors; And I, amongſt the reſt, did adventure to Caſt in my Mite upon that Subject, in a* Compendium *of* Algebra, *Publiſhed* Anno 1695. *which appears to be ſo well Approved of, that Doctor* John Harris *hath Tranſcribed and Inſerted that whole Diſcourſe into his* Lexicon Technicum; Or, The Univerſal Engliſh Dictionary. *Since then I wrote ſomewhat fuller upon the ſame Subject, in the Algebraick Part of the* Young Mathematician's Guide, *Publiſh'd* Anno 1707.

But even that, and all that I've hitherto ſeen upon this Subject, ſeems to fall ſhort of what might be deſired: For moſt Authors, which treat of Intereſt *and* Annuities, &c. *do perform their Computations by the Help of particular Tables, Calculated to ſeveral Rates of* Intereſt *and* Diſcompt.

And thoſe Authors which perform their Computations in Compound Intereſt, &c. *by* Logarithms, *ſeem (to me) to be too general and ſhort in their Theorems, by which thoſe Calculations are perform'd, eſpecially in reſpect of the different Times of Payment,* &c.

And what I've heretofore done has been by Algebraick Theorems, to be perform'd either by the Pen, or with the help of a ſmall Table, Calculated on purpoſe; which was indeed rather to ſhew what may poſſibly be perform'd by the Pen (without a great many Tables of ſeveral Rates) *than for common Practice: Nor can it be ſuppoſed, that every Reader, which may have Occaſion to peruſe the Buſineſs of* Intereſt, *is fitly qualified to underſtand thoſe Theorems ſo well, as to apply them to Practice.*

And for that Reaſon I was requeſted by an Ingenious Gentleman (to whom I ſtand highly Oblig'd) to Explain thoſe Theorems, and Reduce them into Practical Rules, expreſs'd in Words at length, that ſo they might become of general Uſe; the which I've accordingly done along with the Theorems, ſo plainly, that (I preſume) any One, who is but a little verſ'd in the firſt Four Rules of Arithmetick, *may be able to Reſolve all the various Caſes relating to* Intereſt, *both* Simple *and* Compound, *with thoſe of* Annuities, *or* Leaſes, &c. *for any propoſed Time, and at any given Rate of* Intereſt, *both by the Pen, and a ſmall Table of* Logarithms *hereunto annex'd. Alſo I've added a New Method of finding the true Equated Time, for the Diſcharging of ſeveral Sums at one entire Payment, without Loſs either to the Debtor, or Creditor; a thing which hath hitherto been imperfect.*

And in order to render the whole Work as plain and eaſie to be underſtood as poſſibly I can, I've given Two or Three Examples in every Caſe, for different Rates of Intereſt, *and Times of* Payment, *with their Operations at large: For altho' Rules are never ſo well expreſſed in Words, (according to the Author's Senſe) yet there may (and perhaps will) ariſe ſome ſeeming Difficulties in them, which Examples will help to Explain and render Eaſie.*

To be brief; When this Tract was peruſed in Manuſcript by a Perſon that has very good Skill in the Buſineſs of Intereſt, &c. *it gave ſuch ample Satisfaction, that I was prevail'd with to Publiſh it; and accordingly I've adventured to ſend it Abroad into the World, in hopes it will be found uſeful, which is the chief thing deſir'd.*

London, Novemb. *By*
21th, 1709.

J. WARD.

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A
KEY to INTEREST,
William BOTH Mycock
Simple and *Compound*, &c.

1730

CHAP. I.

Of Decimal Arithmetick.

I Do here take it for granted, that the Reader is well acquainted with *Vulgar Arithmetick*, eſpecially the common Rules; *viz.* *Addition*, *Subſtraction*, *Multiplication*, and *Diviſion of whole Numbers*: But becauſe the Computations of *Intereſt*, and *Values* of *Annuities*, &c. cannot well be perform'd by *Logarithms*, or indeed by any other Method whatſoever, without ſome Knowlege of thoſe Rules in *Decimal Fractions* or *Parts*, which perhaps he is not ſo well acquainted with; I therefore thought it convenient to give a brief Account thereof.

Sect. 1. Notation of *Decimals*.

Any thing which is called One, (whether it be *Coin*, *Weight*, *Meaſure*, or *Time*, &c.) as one *Foot*, one *Yard*, one *Pound*, one *Shilling*, one *Year*, or one *Day*, &c. we conceive it to be *Divided* into Ten equal Parts, and every one of thoſe Parts, are ſuppoſed to be ſub-divided into other Ten equal Parts; and ſo on by a *Decimal* Diviſion (*viz.* by Tens) *ad infinitum*.

The *Unit* or 1, being thus divided by Imagination into 10 . 100 . 1000 . or 10000 *&c.* equal Parts at pleaſure, any *Number* of thoſe Parts may be as eaſily expreſs'd and ſet down as whole *Numbers*, they being only diſtinguiſh'd and known from whole Numbers, by a *Comma*, or *Point*, as in the following Table.

Whole Numbers.							*Decimal Parts.*						
	5	4	3	2	1	0,	1	2	3	4	5	6	
&c.	*Hundreds of Thouſands*	*Tens of Thouſands*	*Thouſands*	*Hundreds*	*Tens*	*Unit's Place*	*Parts of Ten*, or $\frac{1}{10}$	*Parts of a Hundred*	*Parts of* 1000	*Parts of* 10000	*Parts of* 100000	*Parts of* 1000000	&c.

From this Table it does plainly appear,

1. That, as whole Numbers do increaſe or become greater from the *Unit*'s Place towards the Left Hand, by a ten-fold Proportion: ſo *Decimal* Parts do decreaſe or become leſs from the *Unit*'s Place towards the Right Hand, in the ſame Proportion, *viz.* by *Tens*.

2. That the *Decimal* Parts are only ſeperated and known from whole Numbers by a *Point*, or *Comma*; and take their Denomination from their Diſtance below the Unit's Place towards the Right Hand.

Thus { 0,5 is 5 parts of *Ten*.
0,25 is 25 parts of a *Hundred*.
0,657 is 657 parts of a *Thouſand*, &c.

3. *Cyphers* being annex'd to *Decimal* Parts, alter not their *Values*; That is, thoſe *Cyphers* do neither increaſe nor decreaſe the Value of the Parts they are annexed to.

Thus 0,5 0,50 0,500 0,5000 *&c.* are each of them but five Tenth parts of a Unit or 1.

4. But *Cyphers* prefixed before *Decimal* Parts, do decreaſe their *Value*, by removing them further from the *Comma*, or *Unit*'s place.

Thus { 0,5 is 5 parts of *Ten*.
0,05 is 5 parts of a *Hundred*.
0,005 is 5 parts of a *Thouſand*, &c.

Conſequently the true *Value* of all *Decimal* Parts is eaſily known by their diſtance from the *Unit*'s place (as above :) Which being well underſtood, the following *Rules* will be found very eaſie.

Sect. 3. Addition and Subſtraction of *Decimals*.

Having ſet down all the propoſed Numbers in their reſpective places; *viz*. every *Figure*, as well of the *Decimal* Parts, as of the whole *Numbers*, directly underneath thoſe of the ſame *Value* (or Name;) which may be very eaſily done, if the *Comma's*, or ſeparating *Points*, are carefully placed directly one under another.

Then { Add, or Subſtract them as if they were all whole Numbers; and from their Sum, or Difference, Cut off, by the ſeparating Comma, ſo many places of Decimal Parts, as there are in any of the given Numbers.

Examples in Addition.

Let it be required to find the *Sum* of 43,25 and 2,45 and 29,9 and 7,054 Theſe Numbers being rightly ſet down will ſtand

Thus 43,25
2,45
29,9
7,054
Sum 82,654

Again 378,0009
59,025
470,9
92,0741 } Add
Sum 1000,0000

Examples in Subſtraction.

From	753,25	From	75,094	From	543,000
Take	469,25	Take	68,596	Take	267,546
Rem.	284,00		6,498		275,454

Note, *Addition* and *Subſtraction* do mutually prove each other, as in whole *Numbers*; of which I ſuppoſe it needleſs to inſert Examples.

Sect. 3. Multiplication of *Decimals.*

Whether the *Factors* (*viz.* the Numbers) propoſed to be *Multiplied* together, are either all *Decimal* Parts, or *Decimals* joyned to whole *Numbers*, *Multiply* them together as if they were all whole Numbers, and for the true *Value* of their *Product*, obſerve this *Rule*.

Rule *Cut off* (viz. *ſeparate) with a Comma, ſo many places of Decimal parts in the Product as there are in both the Factors counted together.*

Examples.

Suppoſe it were requir'd to *Multiply* 21,3 with 4,56

The given Factors	21,3		Or	4,56
	4,56			21,3
	1278			1368
	1065			456
	852			912
Product	97,128			97,128

Again, Let it be requir'd to *Multiply* 4573 *into* 7,546 : And 36,4078 *into* 549,3 The Work may ſtand

Thus	4573	*Multiply*	36,4078
	7,546		549,3
	27438		1092234
	18292		3276702
	22865		1456312
	32011		1820390
	34507,858	*Products*	19998,80454

Note, It ſometimes happens, that in *Multiplying* Parts with Parts only, there will not be ſo many *Figures* in their Product as there ought to by places of Parts by the *Rule:* In that Caſe you muſt ſupply the Defect by *prefixing Cyphers* to their Product; As in theſe following

Examples.

0,2365	} *Multiply* {	0,03472
0,2435		0,02364
11825		13888
7095		20832
9460		10416
4730		6944
0,05758775	*Products*	0,0008207808

Sect. 4. Diviſion of *Decimals.*

Diviſion of *Devimals* is perform'd in all reſpects like that of whole *Numbers*; And for diſcovering the true *Value* of the *Quotient*, obſerve this General *Rule.*

Rule { *The places of Decimal parts in the Diviſor and Quotient being counted together, muſt always be equal in Number with thoſe in the Dividend.*

This General Rule admits of Four *Caſes.*

Caſe 1. When the Places of *Decimal* Parts in the *Diviſor* and *Dividend* are equal in Number, then the *Quotient* will be all whole Numbers; As in the Two following Examples.

Dividend.

Diviſor 7,54) 7140,38 (947 the *Quotient.*

6786

3543

3016

5278

5278

(0)

Again, If it were required to Divide 2,6925 by 0,0075

Dividend.

Diviſor 0,0075) 2,6925 (359 *Quotient.*

Caſe 2. When the Number of places of Decimal parts in the Dividend, exceed thoſe in the Diviſor, *Cut off* the Exceſs for Decimal parts in the Quotient.

Examples.

7,54) 71,4038 (9,47 And 0,75) 2,6925 (3,59

Caſe 3. When there is not ſo many places of Decimal parts in the Dividend, as there is in the Diviſor, then annex Cyphers to the Dividend, to make them equal, and the Quotient will be all whole Numbers ; As in *Caſe* 1.

Examples.

Let us ſuppoſe it were requir'd to Divide 1565,7 by 3,684 : And 3615 by 5,784 the Work muſt ſtand thus,

3,684) 1565,700 (425 And 5,784) 3615,000 (625

Caſe 4. If, after Diviſion is finiſh'd, there are not ſo many Figures in the Quotient, as there ought to be Places of Decimal parts, by the General *Rule*, you muſt then ſupply their Defect, by *prefixing* Cyphers before the Quotient Figures.

Examples.

Let it be requir'd to Divide 3,5532 by 756

756) 3,5 5 3 2 (47
3 0 2 4
5 2 9 2
5 2 9 2
(0)

Here Diviſion being finiſh'd, there are but *Two* Figures in the Quotient, whereas there ought to be Four places of Decimal parts, by the General *Rule* ; therefore two Cyphers muſt be prefix'd before the Quotient Figures, thus 0,0047

That is, 756) 3,5532 (0,0047 is the true Quotient required.

Suppoſe it were required to Divide 0,0007475 by 0,575; *viz.* all Decimal parts; Then It will be 0,575) 0,0007475 (0,0013 the Quotient, &c.

Note, *Multiplication* and *Diviſion* do mutually prove the Truth of each other: That is, If in *Multiplication* you Divide the Product by either of the Factors, the Quotient will be the other Factor, if the Work be true.

So in *Diviſion*, If you Multiply the Quotient and the Diviſor together, their Product (*being Added to what remain'd after Diviſion*) will be the Dividend, if that Work be true.

Sect. 5. To **Reduce** (or rather **Change**) *Vulgar Fractions* into *Decimal* Parts, and the contrary.

Rule { *Annex Cyphers to the Numerator of the given Fraction; then Divide it by the Denominator, and the Quotient will be the Decimal parts ſought.*

Examples.

Suppoſe it were requir'd to *Reduce* $\frac{3}{4}$ into Decimal parts: Then it will be, 4) 3,00 (0,75 the Decimal parts required;
And $\frac{1}{2}$ is 0,5 Thus 2) 1,0 (0,5 the Decimal;
Or $\frac{1}{4}$ is 0,25 Thus 4) 1,00 (0,25 the Decimal.

And thus may the Decimal parts *equivalent* to any known part, or parts of *Coin*, *Weight*, *Meaſures*, or *Time*, &c. be eaſily found, if you firſt reduce the given parts of the *Coin*, or *Time*, &c. into a *Vulgar Fraction*, whoſe Denominator is the Number of thoſe known parts contain'd in the Integer, and the given parts its Numerator.

Examples in Coin.

Suppoſe it were requir'd to find the Decimal parts of a Pound Sterling, equal to 17 *s.* 6 *d.*

Firſt, becauſe 20 *s.* makes one Pound, therefore 17 *s.* is $\frac{17}{20}$ of a Pound; and becauſe there's 240 Pence in a Pound, therefore 6 *d.* is $\frac{6}{240}$ of a Pound.

Then 20) 17,00 (0,85 is the Decimal of 17 *s.*
And 240) 6,000 (0,025 is the Decimal of 6 *d.*

Conſequently their Sum 0,875 will be the Decimal parts equivalent to 17 *s.* 6 *d.* as was requir'd, &c.

Example in Time.

Let it be requir'd to find the Decimal parts equal to 126 Days.

Firſt, becauſe 365 Days do make a common Year, therefore 126 Days is $\frac{126}{365}$ parts of a Year.

Then 365) 126,0000 (0,3452 will be the Decimal parts equivalent to 126 Days, &c.

The like is to be underſtood in *Reducing*, or *Changing* the known parts of any propoſed Integer into Decimal parts, equal to thoſe known parts; or at leaſt ſo near the Truth, as may be thought neceſſary to approach.

Now the contrary Operations to theſe, *viz.* to find the *Value* of any given Decimal parts of *Coin*, or *Time*, &c. is only the *Converſe* of the Work above, and may be perform'd by this following Rule.

Rule { *Multiply the given Decimal parts with ſuch a Number of Units as are contain'd in the Integer to which the given Decimal does belong, and the Product will be the Value of the ſaid Decimal.*

Examples.

1. Let 25,7875 ſignifie 25 *l.* and the Decimal parts of 1 *l.* How many Shillings and Pence do theſe Decimal parts denote? That is, How many Shillings and Pence are equal to 0,7875 Decimal parts of a Pound *Sterling.*

Firſt, 25,7875
Multiplied with 20 the Shillings in 1 l.
Gives Shillings 15,7500 and the Decimals of 1 s.
Multiply with 12 the Pence in 1 s.
150
75
Gives Pence 9,0000 Hence the Anſwer is, That 25 l. 15 s. 9 d. is the ſame with 25,7875 Decimals of *Coin.*

2. Suppoſe the ſame 25,7875 to ſignifie 25 Years, and the Decimal parts of one Year; How many *Days*, *Hours*, &c. would thoſe Decimal parts denote?

Anſwer, 25 *Years*, 287 *Days*, 10 *Hours*, and 30 *Min.*

Thus 25,7875
Multiplied with 365 the Days in one Year,
39375
47250
23625
Gives Days 287,4375 and Decimal parts of 1 Day,
Multiply with 24 the Hours in 1 Day.
17500
8750
Gives Hours 10,5000 and 0,5 parts of 1 Hour,
Multiply with 60 the Minutes in 1 Hour.
Minutes 30,0000

And thus may any propoſed Number of Decimal parts be *Reduced* or *Changed* into the known parts of what they repreſent, *viz.* whether they be parts of *Coin*, *Weight*, *Meaſures*, or *Time*, &c. a due regard being had to the Number of Units which are contain'd in the Denomination of the thing to which the Decimal Parts belong.

CHAP. II.

The Definition and Uſe of Logarithms in General.

Sect. 1. Definitions.

Logarithms are a ſort of Artificial Numbers, ſo adapted to correſpond with Natural Numbers, that the *Addition* and *Subſtraction* of them, do exactly anſwer to the *Multiplication* and *Diviſion* of thoſe Natural Numbers they are adapted to.

That is, If any two given Numbers are either to be *Multiplied*, or *Divided*, the Logarithms of thoſe Numbers being accordingly *Added*, or *Subſtracted*, their *Sum*, or *Difference* will be the Logarithm of that Natural Number, which is the Product, or Quotient of ſuch *Multiplication*, or *Diviſion*.

And that the Value of any Product, or Quotient, ſo found by the Sum, or Difference, of two Logarithms may be truly known, every Logarithm of the prime Numbers, *viz.* of 1 . 10 . 100 . 1000 *&c.* in whole Numbers; And of 0,1 . 0,01 . 0,001 0,0001 *&c.* in Decimal parts, is the *Index* or *Characteriſtick* to all the intermediate Logarithms which are between them: As in this

TABLE.

Whole Num.	*Logarithms*	*Decim. parts*	*Logarithms*
1	0.000000	1	0.000000
10	1.000000	0,1	$\bar{9}$.000000
100	2.000000	0,01	$\bar{8}$.000000
1000	3.000000	0,001	$\bar{7}$.000000
10000	4.000000	0,0001	$\bar{6}$.000000
&c.	&c.	&c.	&c.

This *Table* ſhews ſome of the principal *Logarithms*, or *Characteriſticks*, both of whole Numbers, and of Decimal parts, with ſo many Cyphers annex'd to each of them (*like Decimal parts*) as the number of places are in the following *Table* of *Logarithms*: From whence it will be eaſie to perceive, That,

If 0, be made the Logarithm of 1. And 1, be made the Logarithm of 10. Then, all the intermediate Logarithms which belong to the natural Numbers, between 1, and 10, (viz. *the Logarithms of* 2 . 3 . 4 . 5 . 6 &c.) muſt needs be Decimals leſs than 1. And for the ſame reaſon, That,

If 1. be the Logarithm of 10. And 2. be made the Logarithm of 100. Then all the intermediate Logarithms, which belong to the natural Numbers between 10. and 100. will be a 1, with Decimal parts annexed to it: And conſequently all the Logarithms of the natural Numbers between 100, and 1000, will be a 2, with Decimal parts annexed to it; and ſo on for higher Numbers, as in the *Table*.

Now all theſe principal Logarithms, *viz*. 0 . 1 . 2 . 3 . 4 &c. of whole Numbers, are call'd *Affirmative Indices*, or *Characteriſticks*.

And thoſe which belong to Decimal parts or *Fractions*, *viz*. $\bar{9}$. $\bar{8}$. $\bar{7}$. $\bar{6}$ &c. are call'd *Negative Indices* or *Characteriſticks*; being diſtinguiſh'd from the Affirmative Indices by the Negative Sign — ſet over their Heads.

For, If 0, be the Logarithm of 1. (*as above*) Then it follows, that all the Logarithms of Fractions (viz. *of Numbers leſs then* 1.) muſt needs be leſs than 0. That is, they muſt be *Negative* Numbers.

From what hath been here ſaid, it will be eaſie to deduce the General *Rule*, by which the Diſtance of the natural Numbers from the Unit's place is always known.

Viz. { *That every Index or Characteriſtick is leſs by an Unit or* 1. *than the Number of places of Figures in the natural Numbers to which it belongs.*

As for inſtance in the following *Numbers,* wherein I ſuppoſe 5381 to be a whole Number; whoſe Logarithm is 3.730863 Now, if this Natural Number 5381. be otherwiſe taken or varied in its places, then the *Logarithms* will ſtand thus:

Natural Numbers	*Logarithms.*
5381	3.730863
538,1	2.730863
53,81	1.730863
5,381	0.730863
0,5381	$\bar{9}$.730863
0,05381	$\bar{8}$.730863
0,005381	$\bar{7}$.730863 &c.

In theſe Examples you ſee, that the Logarithms are all the ſame, ſave only the *Indices* or *Characteriſticks,* are alter'd according to the Diſtance of the firſt Figure of the natural Number from the Unit's place; which being once well underſtood, it will be eaſie to find the Logarithm of any given Number in the *Table* of *Logarithms,* and to prefix its proper *Index* to it.

Or, if any Logarithm, with its Index be given; To find its correſpondent Number, ſo far as the *Table* of *Logarithms* extends; and upon occaſion to one or two places further, with a very ſmall trouble.

Sect. 2. To find the **Logarithm** of any given *Number.*

The firſt Page of the annex'd *Tables* of *Logarithms* contains all the Natural Numbers in their proper Order from 1. to 100. And againſt every one of thoſe Numbers is placed its *Logarithm,* with its *Index* or *Characteriſtick* before it.

Thus, againſt the Number 28 is its *Log.* 1.447158 And againſt the Number 89 is its *Log.* 1.949390. And ſo on for the reſt.

In the firſt Column of all, the following Pages (*under Num.*) the natural Numbers proceed in their due Order from 100. To 1000. And in the next Column (*uuder* 0) againſt every one of thoſe Numbers, is the Decimal Part of its Logarithm, without any Index; to which you muſt prefix its proper Index, according as the natural Number you make uſe of requires.

As for inſtance; Againſt the Number 856 (*under* 0) is 932474 To which if 2 the Index of 856 be prefix'd, it will become 2.932474 the compleated Logarithm of 856.

The other five Columns of each Page contains the Logarithms of all the Numbers from 1000 To 10000 Thoſe in the Left Hand Pages are diſtinguiſh'd on the Top of the Columns with the Figures 0 . 1 . 2 . 3 . 4 . And thoſe in the Right Hand Pages with 5 . 6 . 7 . 8 . 9. So that to find the Logarithm of any Number between 1000 and 10000 : As ſuppoſe of 5468, you muſt look for the Three firſt Figures, *viz.* 546 in the firſt Column under *Num.* and for the laſt Figure, *viz.* 8 at the Top: Then in the Column under the laſt Figure 8 and right over-againſt the Three firſt Figures 546 there is 737829 To which if 3 the Index of 5468 be prefixed, you will then have 3.737829 the compleated Logarithm of 5468 as was required: And ſo for any other Logarithm of any propoſed Number not exceeding 10000.

But if the propoſed Number be above 10000 (*which is the Limits of the annexed Table*) Then the Logarithm of that Number muſt be found by help of the Common Difference of the Logarithms, which is in the laſt Column of every Page under *Diff.*

Thus,

Find the Logarithm of the firſt Four Figures of the given Number, without its Index, (as above) and Multiply the Common Difference which ſtands againſt that Logarithm (under Diff.*) with the other Figures of the given*

Number, Caſting off ſo many Figures of that Product as there are in the Multiplicator.

Then Add the Remaining Figures of that Product, to the Logarithm of the firſt Four Figures, and to their Sum prefix the proper Index, and you will have the compleated Logarithm required.

Example.

Suppoſe it were required to find the Logarithm of 698476. Firſt the Logarithm of 6984 is found in the Table (*as above*) to be 844104, and againſt it, under *Diff.* is 62. This 62 being Multiplied with 76 (*the other Two Figures of the given Number*) produces 4712. *Cut off* the 12 (viz. *the Two laſt Figures*) and then *Add* the 47 to the *Logarithm* laſt found, and the *Sum* will be 844151; to which prefixing 5 the proper Index of the given Number 698476, it will be 5.844151 the Logarithm of 698476 as was required.

Sect. 3. To find the Number to any given Logarithm.

Omit the Index or Characteriſtick, of the given Logarithm, and then ſeek it in the Table of Logarithms; and if it can be exactly found there, then the Number in the firſt Column (*under Num.*) with that on the Top over the Logarithm, will be the Number required. But if the given Logarithm (*without its Index*) cannot be exactly found in the Table, then the proper Number agreeing to that Logarithm, may be found by the help of the Common Difference of the Logarithms.

Thus, *From the given Logarithm Subſtract the next Leſs, and to the Remainder Annex Cyphers; and then Divide it by the Common Difference found againſt the next leſs Logarithm, under* Diff. *and the Quotient will be a Number that muſt be Annexed to the Number already found againſt the next leſs Logarithm, according as the Index of the given Logarithm denotes.*

Example.

Suppoſe 5.660279 were a given Logarithm, and it were required to find the natural Number anſwering to it.

Here the Number ſought muſt conſiſt of Six Places of Figures in whole Numbers, as appears by its Index 5. Which being omitted, I ſeek in the Table of Logarithms for 660279 but not finding it exactly there, I take the next Leſs to it, *viz.* 660201, which ſtands under 3, and againſt 457. Therefore I conclude, that the firſt Four Figures of the Number ſought, muſt be 4573; and the Common Difference found againſt 660201 under *Diff.* is 95.

Then from the given Logarithm 660279
I Subſtract the next Leſs, *viz.* 660201
And there remains 78.

To which Annexing Two Cyphers, (*becauſe there is yet wanting two Places of Figures*) it will become 7800: The which being Divided by the Common Difference 95 the Quotient will be 82. Thus 95) 7800 (82 which muſt be annexed to 4573 (*found before*) and the Sum will be 457382, the Number that anſwers to the given Logarithm 5.660279 as was requir'd

Thus one may, without much Trouble, find the Logarithm of any given Number (*very near*) altho' it exceed the Limits of the Table by 1, 2, or 3 Places of Figures; And alſo the Number Agreeing to any given Logarithm, without the help of ſuch a Table of *Proportional Parts* as is uſually inſerted along with the Table of Logarithms for that Purpoſe.

Sect. 4. To perform Multiplication by Logarithms.

The Multiplication of any Two given Numbers together, may be perform'd by Logarithms, Thus, { *Add the reſpective Logarithms of the given Numbers together, and their Sum will be the Logarithm of the Product required; due regard being had to the true ordering of their Indices; which admits of Three Caſes.*

Caſe 1. If the Indices are both Affirmative, their Sum, with what ariſes from the Addition of their Logarithms, will be an Affirmative Index.

Examples.

	Numbers.		*Logarithms.*	
Multiplicand	7564	——	3.878752	} Add
Multiplier	75	——	1.875061	
Product	567300	——	5.753813	

Again,

Multiplicand	75,64	——	1.878752	} Add
Multiplier	7,5	——	0.875061	
Product	567,300	——	2.753813	

Caſe 2. If one of the Indices be Affirmative, and the other be Negative; and if their Sum be above 10. Then Caſt off 10. and the Remainder will be an Affirmative Index: But if their Sum be Leſs than 10. it will be a Negative Index.

Examples.

	Numbers.		*Logarithms.*	
Multiplicand	75,64	——	1.878752	} Add
Multiplier	0,75	——	$\overline{9}$.875061	
Product	56,7300	——	1.753813	

Again,

Multiplicand	75,64	——	1.878752	} Add
Multiplier	0,0075	——	$\overline{7}$.875061	
Product	0,5673	——	$\overline{9}$.753813	

Caſe 3. If both the Indices are Negative, and their Sum be above 10. Then Caſt off 10. and the Remainder will be a Negative Index. If their Sum be juſt 10 Add 1 to it. If it be under 10, Add 10 to it, and that Sum will be a Negative Index.

The two Laſt Parts of this *Caſe* ſeldom come into practice. I ſhall therefore only give an Example of the Firſt.

Example.

	Numbers.		*Logarithms.*	
Multiplicand	0,0347	——	$\bar{8}$.540329	Add
Multiplier	0,0236	——	$\bar{8}$.372912	
Product	0,00081892	——	$\bar{6}$.913241	

Sect. 5. Diviſion perform'd by Logarithms.

To Divide one Number by another, is only the Converſe of the Laſt Work, and is perform'd by Logarithms,

Thus, *Subſtract the Logarithm of the Diviſor from the Logarithm of the Dividend, and the Remainder will be the Logarithm of the Quotient; with this Conſideration; That*

When a Leſſer Number Divides a Greater, the Index of the Quotient-Logarithm will be Affirmative: But if a Greater Number Divides a Leſſer, then the Index of the Quotient-Logarithm will be Negative, and there muſt be 10 Added to the Index of the Logarithm of the Dividend, when Subſtraction cannot be made without it.

Examples.

	Numbers.		*Logarithms.*	
Dividend	567300	——	5.753813	Sub.
Diviſor	7564	——	3.878752	
Quotient	75	——	1.875061	

Again,

Dividend	68,6	——	1.836324	Sub.
Diviſor	78,4	——	2.894316	
Quotient	0,0872	——	$\bar{8}$.942008	

Again,

	Numbers		Logarithms.	
Dividend	56,73	—	1.753813	} Sub.
Diviſor	0,75	—	9.875061.	}
Quotient	79,64	—	1.878752	

Sect. 6. To *Extract* the Square and Cube Roots, *&c.* by Logarithms.

If the Logarithm of any given Number be Divided by 2, the Quotient will be the Logarithm of the *Square Root* of that Number. And if the Logarithm of any Number be Divided by 3, the Quotient will the Logarithm of the *Cube Root* of that Number: And ſo on for higher Powers.

Examples.

Suppoſe it were required to Extract the *Square Root* of 6968, whoſe Logarithm is 3.843108.

Then 2)3.843108(1.921554 This Quotient is the Logarithm of 83,4745 &c. which is the *Square Root* of 6968, as was required.

Again; Let it be required to Extract the *Cube Root* of 6968, whoſe Logarithm is 3.843108 as before.

Then 3)3.843108(1.281036 This Quotient is the Logarithm of 19,1 &c. which is the *Cube Root* of 6968, As was required.

And in the ſame manner the *Biquadrat Root* or that of the Fourth Power, may be found; if the Logarithm of the given Number be Divided by 4. And if the Logarithm of any given Number be Divided by 5, the Quotient will be the Logarithm of the *Surſolid Root*, or that of the 5th Power, and ſo on for the 6th, 7th, 8th, or any propoſed Root of a ſingle Power, how high ſoever it be, provided the Index of the Logarithm of the given Number be Affirmative. But, The Dividing of Logarithms, whoſe Indices are Negative, (by 2 . 3 . 4 . 5 . &c.) and to determine the true Quotient-Index hath been thought ſo difficult a Work, that the *Learned* and *Ingenious* Mr. *Oughtred* contrived a Table on purpoſe to perform it; *Vide Key of the Mathematicks*, pag. 168.

Now according to the Negative Indices I have here made uſe of, that Work may be very eaſily perform'd.

Thus { *If you are to Divide a Logarithm that hath a Negative Index by* 2, *Add* $\overline{10}$ *to the given Index: If by* 3, *then Add* $\overline{20}$ *to the given Index: If by* 4, *then Add* $\overline{30}$ *to the given Index*, &c.

That is, Still increaſing the Index of the given Logarithm with $\overline{10}$, as the Diviſor doth increaſe by a Unit or 1.

Examples.

Suppoſe it were required to Extract the *Square Root* of 0,054756 whoſe Logarithm is $\overline{8}.738432$.

Then 2)$\overline{18}.738432$($\overline{9}.369216$ This Quotient is the Logarithm of 0,234 which is the *Square Root* of 0,054756 as was requir'd.

Again, Let it be requir'd to Extract the *Cube Root* of 0,054756 whoſe Logarithm is $\overline{8}.738432$ as before.

Then 3)$\overline{28}.738432$($\overline{9}.579477$ This is the Logarithm of 0,3797 &c. which is the *Cube Root* of 0,054756 as was requir'd.

Once more; Suppoſe it were requir'd to Extract the Second *Surſolid Root* or that of the Seventh Power, out of 0,054756 whoſe Logarithm is $\overline{8}.738432$ as above.

Then 7)$\overline{68}.738432$($\overline{9}.819776$ This is the Logarithm of 0,66035 &c. which is the *Root* requir'd, &c.

Thus far may ſuffice concerning the Nature and Uſe of the Table of Logarithms in general; which being a little conſider'd of, it will be eaſie to Apply them to the following *Calculations*.

CHAP. III.

The Calculation of Queſtions in Simple Intereſt and Annuities; perform'd both by the *Pen*, and by *Logarithms*.

BEfore we proceed to the following Computations, it may be convenient to premiſe a few uſeful things, that will help to Explain and Shorten the Work. And firſt, Of a few common *Characters*; *Viz*.

+	Signifies *More*, or the Affirmative Sign of *Addition*.
—	Signifies *Leſs*, or the Negative Sign of *Subſtraction*.
×	Signifies *Into* or *With*, and is the Sign of *Multiplication*.
=	Signifies *Equal* to, or the Sign of *Equality*.
Log.	Denotes the Compleated *Logarithm* of any Number.

And when the *Ratio* of the *Rate* of *Intereſt* is mention'd, it ſignifies only the Simple Intereſt of 1 *l.* for one Year, at any propoſed Rate of Intereſt *per Cent.* which may be thus found by the *Rule of Three*.

As 100 : Is to 6 :: So is 1 : To 0,06 the *Ratio* of the Rate of 6 *per Cent. per Annum*.

Or, As 100 : Is to 7 :: So is 1 : To 0,07 the *Ratio* of the Rate of 7 *per Cent. per Annum*, &c.

The which may alſo be found by Logarithms.

Thus, { *From the Logarithm of the given Rate of Intereſt, Subſtract the Logarithm of* 100 (viz. 2.000000) *and the Remainder will be the Logarithm of the Ratio of that Rate.*

Example.

Suppoſe the given Rate of Intereſt to be that of 6 *per Cent. per Annum* : Then
The Logarithm of 6 is 0.778151 } Subſtract
The Logarithm of 100 is 2.000000 }
The Remainder is the Log. $\bar{8}$.778151 of 0,06
That is, 0,06 is the *Ratio* of 6 *per Cent.* &c.

And thus may the *Ratio* of any other propoſed Rate of *Intereſt per Cent.* be eaſily obtain'd.

But becauſe 'tis the Logarithms of thoſe Ratio's, that are of uſe in the following Calculations relating to Simple Intereſt, I have here annexed a ſmall Table of ſeveral Rates of Intereſt, with their Ratio's, and the Logarithms of thoſe Ratio's.

Rates of Intereſt perCent.	*Ratio's of thoſe Rates.*	*Logarithms of thoſe Ratio's.*	*Rates of Intereſt perCent.*	*Ratio's of thoſe Rates.*	*Logarithms of thoſe Ratio's.*
3	0,03	$\bar{8}$.477121	7	0,07	$\bar{8}$.845098
4	0,04	$\bar{8}$.602060	7½	0,075	$\bar{8}$.875061
4½	0,045	$\bar{8}$.653212	8	0,08	$\bar{8}$.903090
5	0,05	$\bar{8}$.698970	8½	0,085	$\bar{8}$.929419
5½	0,055	$\bar{8}$.740363	9	0,09	$\bar{8}$.954242
6	0,06	$\bar{8}$.778151	9½	0,095	$\bar{8}$.977724
6½	0,065	$\bar{8}$.812913	10	0,1	9.000000

Theſe things being premiſed, we may proceed to the following Work; and firſt of Money forborn at any Rate of Simple Intereſt, &c.

Sect. 1. Of Simple Intereſt.

In the *Young Mathematician's Guide*, pag. 245. (from whence the following Rules are deduc'd) I've made uſe of Letters to denote the ſeveral Parts of the Queſtion,

Viz.
P. Signifies any Principal or Sum put to Intereſt.
T. The Time of its continuing at Intereſt.
R. The Ratio of the Rate of Intereſt *per Cent.*
A. The Amount of the Principal and its Intereſt.

Any three of theſe Parts being given, the other may be found by help of this General

Theorem, $TRP + P = A$.

This Theorem admits of four Caſes or Variety of Queſtions.

Caſe 1. If *P*, *T*, and *R*, are given, thence to find *A*. That is, If any Principal, with the Time of its being at Intereſt, and the Rate of Intereſt *per Cent. per Annum* are given, To find the Intereſt, and the Amount.

This Queſtion I take to be of the moſt general Uſe of any that Occurs in the whole Buſineſs of Simple Intereſt; and may be perform'd thus:

First by Common *Arithmetick.*

Rule. *Multiply the Principal, the Time, and the Ratio of the Rate, all three together; and their Product will be the Intereſt: To which Add the Principal, and the Sum will be the Amount requir'd.*

Example.

What Sum will 567 l. 10 s. *Amount to in Nine Years, at the Rate of* 6 per Cent. per Annum?

Here is given $P = 567{,}5$ $T = 9$ and $R = 0{,}06$ To find *A*; which by the Rule is done thus:

Firſt

Firſt $P =$	567,5	*Multiply.*	Again,	5107,5	*Multiply.*
And $T =$	9		$R =$	0,06	
Product	5107,5			306,45 = 306 *l.* 9 *s.*	

That is, 306 *l.* 9 *s.* is the Intereſt of 567 *l.* 10 *s.* for Nine Years: Then 306 *l.* 9 *s.* + 567 *l.* 10 *s.* = 873 *l.* 19 *s.* = A, the Amount requir'd.

The ſame perform'd by *Logarithms*.

Thus, *To the Log. of the Principal, Add the Log. of the Time, and the Log. of the Ratio; their Sum will be the Log. of the Intereſt; to which Add the Principal,* &c. *As above.*

That is, in the ſame *Example*.

Thus, $P = 567{,}5$	its Log. is	2.753966	Add
And, $T = 9$	its Log. is	0.954242	
$R = 0{,}06$	its Log. is	8.778151	
The Sum is the Logarithm		2.486359	of 306,45

That is, 306 *l.* 9 *s.* is the Intereſt as before, to which the Princip. 567 *l.* 10 *s.* being Added, the Sum is 873 *l.* 19 *s.* = A, the Amount requir'd, as above.

Caſe 2. When A, T, and R, are given; To find P: That is, To find what Principal or Sum, being put to Intereſt any aſſigned Time, will Amount to a propoſed Sum in that Time, at any given Rate of Intereſt *per Cent. per Annum, &c.*

Firſt by the *Pen* only.

Rule. *Multiply the Time with the Ratio of the Rate, and to their Product Add* 1: *Then Divide the propoſed Amount by that Sum, and the Quotient will ſhew the Principal required.*

Example.

What Principal or Sum of Money, being put to Intereſt for Nine Years, will Amount to (*or Raiſe a Stock of*) 873 *l.* 19 s. *at* 6 per Cent. &c.?

Or thus; *Suppoſe a Debt of* 873 l. 19 s. *were not to be paid until Nine Years hence; What would it be worth in ready Money; the Creditor allowing the Rate of* 6 per Cent. *Diſcompt to the Debtor?*

In this Queſtion there is given, $A = 873{,}95$. $T = 9$ and $R = 0{,}06$ To find P,

Thus $T = 9$ } *Mult.* Then $0{,}54 + 1 = 1{,}54$
$R = 0{,}06$ } And $1{,}54) 873{,}95 (567{,}5 = P$.
Product 0,54

That is, $567{,}5 = 567$ *l.* 10 *s.* is the Principal, (*or Ready Money*) requir'd.

The ſame may be perform'd by *Logarithms*,

Thus, { *Add the Log. of the Time to the Log. of the Ratio of the Rate, and to the Number found by their Sum Add* 1. *Then if the Log. of that Sum be Subſtracted from the Log. of the propoſed Amount, there will Remain the Log. of the Principal requir'd.*

That is, in the ſame *Example*,

Thus $T = 9$ its Log. is 0.954242 } Add
And $R = 0{,}06$ its Log. is $\bar{8}.778151$ }
The Sum is this Logar. 9.732393 of 0,45. To which Adding 1, it will become 1,45.

Then $A = 873{,}95$ its Log. is 2.941487 } Subſtract
And 1,54 its Log. is 0.187521 }
Remains the Logarithm 2.753966 of $567{,}5 = P$

That is, 567 *l.* 10 *s.* is the Principal required by the Queſtion, as above.

Caſe 3. Suppoſe P, T, and A, were given; To find R, *viz.* Having any Principal with the Time of its being at Intereſt, and the Sum it's propoſed to Raiſe or Amount to in that Time, given; thence to find the Rate of Intereſt *per Cent. per Annum.*

First

Firſt by the *Pen* only.

Rule. { *If the Difference between the propoſed Amount and the Principal, be Divided by the Product of the Principal Multiplied into the Time, the Quotient will ſhew the Ratio of the Rate of Intereſt requir'd.*

Example.

At what Rate of Intereſt per Cent. *will* 567 l. 10 s. *Raiſe a Stock, or Amount to* 873 l. 19 s. *in Nine Years time?*

Here is given $P = 567{,}5$ $T = 9$, and $A = 873{,}95$ To find R; which by the Rule is

Thus, $873{,}95 - 567{,}5 = 306{,}45$ the *Dividend.*
And $567{,}5 \times 9 = 5107{,}5$ the *Diviſor.*

Then 5107,5) 306,45 (0,06 = R, the Ratio of the Rate of Intereſt;

And As 1 : Is to 0,06 :: So is 100 : To 6 the Rate of Intereſt *per Cent. &c.* as was required.

Or by *Logarithms,*

Thus, { *From the propoſed Amount Subſtract the Principal (as above;) then from the Log. of the Remainder, Subſtract the Sum of the Logarithms of the Principal and the Time, and there will Remain the Log. of the Ratio of the Rate,* &c.

That is, in the ſame Example,

$873{,}95 - 567{,}5 = 306{,}45$ its Log. 2.486359
And $P = 567{,}5$ its Logarith. is 2.753966 } Add
$T = 9$ its Logarith. is 0.954242 }
From the firſt Log. Subſt. this Log. 3.708208
And there remains the Logarith. 8.778151 of 0,06

Which ſhews the Rate of Intereſt to be 6 *per Cent. per Annum*, as before by the Pen.

Caſe 4. Having P, R, and A, given, To find T: That is, To find the Time in which any given Principal will Amount to, or Raiſe any propoſed Sum, the Rate of Intereſt *per Cent.* being alſo given.

First by the *Pen* only.

Rule. *If the Difference between the propoſed Amount and the Principal, be Divided by the Product of the Principal Multiplied into the Ratio of the Rate; the Quotient will ſhew the Time requir'd.*

Example.

In what Time will 567 l. 10 s. *Amount to* (*or Raiſe a Stock of*) 873 l. 19 s. *at* 6 per Cent. per Annum?

In this Queſt. there is given $P = 567,5$ $R = 0,06$ and $A = 873,95$ To find T; which is done thus:

Firſt $873,95 - 567,5 = 306,45$ the Dividend.

And $567,5 \times 0,06 = 34,05$ for the Diviſor.

Then $34,05\,)\,306,45\,(\,9 = T$ the Time; *Viz.* Nine Years will be the Time required.

The ſame perform'd by *Logarithms.*

Thus, *From the propoſed Amount Subſtract the Principal (as before;) Then from the Logarithm of the Remainder, Subſtract the Sum of the Logarithms of the Principal and Ratio of the Rate; and there will Remain the Logarithm of the Time.*

That is, in the Laſt Example;

Thus $873,95 - 567,5 = 306,45$ its Log. 2.486359

$P = 567,5$ its Logarith. is 2.753966 } Add

$R = 0,06$ its Logarith. is 8.778151 }

From the firſt Log. Subſt. this Log. 1.532117

There remains the Logar. of $9 = T$ 0.954242

Which ſhews, that the Time ſought is juſt Nine Years.

If the Work of theſe Four Examples, and the Rules by which they are perform'd, be well underſtood, they will be ſufficient (*notwithſtanding there is really but*

but One Queſtion, only it is varied according to the ſeveral Caſes) to ſhew how any Queſtion of the like kind may be truly Reſolv'd, at any propoſed Rate of Simple Intereſt, and for any aſſign'd Time; eſpecially if the Time given (or ſought) does conſiſt of compleat or whole Years.

But if the Time given (or ſought) does not conſiſt of whole Years, as moſt generally it does not, it being either Leſs than a Year; or Years, and ſome Parts of a Year, as *Weeks*, *Months*, or *Quarters &c.* Then the odd Time, leſs than a compleat Year, muſt be Reduced (or Converted) into Decimal Parts of a Year (*per Sect.* 5. *p.* 7.) And unleſs ſuch Parts of a Year chance to be juſt $\frac{1}{4}$ $\frac{1}{2}$ or $\frac{3}{4}$ of a Year, then the beſt way will be to Reduce the Odd Time into Days, and then work with the Decimal Parts of a Year that are equivalent to thoſe Number of Days.

And for the eaſie and ready finding out of the true Number of Days that are contain'd between any two aſſigned Times leſs than a Year, and the Decimal Parts of a Year that are equal to thoſe Days, I have here inſerted two ſmall Tables.

The Uſe of the following Table of Months and Days, is no more but thus; Find the firſt propoſed Month at the Top or Head of its reſpective Part of the Table; and in the ſame Column under it, look for the other Month, and by it ſtands the Number of Days requir'd, according to the Title of the Table.

As for Example; From the 1ſt, 5th, 10th, 17th, or any other Day in *April*, To the 1ſt, 5th, 10th, 17th (*viz. to the ſame*) Day in *December*, is juſt 244 Days.

Or, From the 4th, 7th, or 12th, *&c.* of *October*, To the 4th, 7th, or 12th, *&c.* of *Auguſt* is juſt 304 Days. And ſo the true Number of Days that are between any two of the ſame Days in the propoſed Months, may be found by Inſpection only.

But if the two given Days of the Month are different, then there Difference muſt be Added to the Number found in the Table: As, ſuppoſe, between the

4th of *October*, and the 25th of *Auguſt*; Here, becauſe the 25th of the one Month, is 21 Days more than the 4th Day of the other Month; therefore the Number of Days requir'd will be 304 + 21; *viz.* 325 Days, and ſo for any other two Months and Number of Days in any propoſed Time throughout the Year; as in this Table.

A Table that ſhews, by Inſpection only, the true Number of Days, from every Day in any Month, to the ſame Day in any other Month, throughout the whole Year.

January	Febru.	March	April	May	June
Feb. 31	Mar. 28	Apr. 31	May 30	June 31	July 30
Mar. 59	Apr. 59	May 61	June 61	July 61	Aug. 61
Apr. 90	May 89	June 92	July 91	Aug. 92	Sept. 92
May 120	June 120	July 122	Aug. 122	Sept. 123	Oct. 122
June 151	July 150	Aug. 153	Sept. 153	Oct. 153	Nov. 153
July 181	Aug. 181	Sept. 184	Oct. 183	Nov. 184	Dec. 183
Aug. 212	Sep. 212	Oct. 214	Nov. 214	Dec. 214	Jan. 214
Sep. 243	Oct. 242	Nov. 245	Dec. 244	Jan. 245	Feb. 245
Oct. 273	Nov. 279	Dec. 275	Jan. 275	Feb. 276	Mar. 273
Nov. 304	Dec. 303	Jan. 306	Feb. 306	Mar. 304	Apr. 304
Dec. 334	Jan. 334	Feb. 337	Mar. 334	Apr. 335	May 334
Jan. 365	Feb. 365	Mar. 365	Apr. 365	May 365	June 365

July	Auguſt	Septem.	Octob.	Novem.	Decem.
Aug. 31	Sept. 31	Oct. 30	Nov. 31	Dec. 30	Jan. 31
Sept. 62	Oct. 61	Nov. 61	Dec. 61	Jan. 61	Feb. 62
Oct. 92	Nov. 92	Dec. 91	Jan. 92	Feb. 92	Mar. 90
Nov. 123	Dec. 122	Jan. 122	Feb. 123	Mar. 120	Apr. 121
Dec. 153	Jan. 153	Feb. 153	Mar. 151	Apr. 151	May 151
Jan. 184	Feb. 184	Mar. 181	Apr. 182	May 181	June 182
Feb. 215	Mar. 212	Apr. 212	May 212	June 212	July 212
Mar. 243	Apr. 243	May 242	June 243	July 242	Aug. 243
Apr. 274	May 273	June 273	July 273	Aug. 273	Sept. 274
May 304	June 304	July 303	Aug. 304	Sept. 304	Oct. 304
June 335	July 334	Aug. 334	Sept. 335	Oct. 334	Nov. 335
July 365	Aug. 365	Sept. 365	Oct. 365	Nov. 365	Dec. 365

A Table for the ready finding the Decimal Parts of a Year, Equal to any Number of Days, &c.

Days.	*Dec. parts*	*Days.*	*Dec. parts*	*Days.*	*Dec. parts*
1	= 0,002740	10	= 0,027397	100	= 0,273973
2	= 0,005479	20	= 0,054794	200	= 0,547945
3	= 0,008219	30	= 0,082192	300	= 0,821918
4	= 0,010959	40	= 0,109589	365	= 1,000000
5	= 0,013699	50	= 0,136986		
6	= 0,016438	60	= 0,164383	¼ *of a Year*	= 0,25
7	= 0,019178	70	= 0,191781	½ *a Year*	= 0,5
8	= 0,021918	80	= 0,219178	¾ *of a Year*	= 0,75
9	= 0,024657	90	= 0,246575		

The Uſe of this Table is thus:

If the propoſed Number of Days can be exactly found in the Table (under *Days*) their Decimal Parts are alſo found againſt them by Inſpection only.

But if the true Number of Days cannot be exactly found there, then both they, and their Decimal Parts muſt be collected out of the Table at twice, or thrice, according as their Number requires.

As for Example: Suppoſe it were required to find the Decimal Parts of a Year equal to 135 Days?

$$\begin{array}{lrl} & \textit{Days.} & \textit{Dec. parts.} \\ \text{Then}\left\{\begin{array}{r}\\ \\ \\ \end{array}\right. & \begin{array}{r}100\\30\\5\end{array} & \left.\begin{array}{l}=0,273973\\=0,082192\\=0,013699\end{array}\right\}\text{ Add all theſe together.}\end{array}$$

Hence 135 = 0,369864 the Decimal Parts required.

And thus may the Decimal Parts of a Year, equivalent to any given Number of Days, be very eaſily found to Six Places of Figures; but for Common Buſineſs it may ſuffice to work with only Four of thoſe Places, and in ſmall Sums, 2, or 3 Places (according to Diſcretion) may be near enough to the Truth.

Or the Decimal Parts, equivalent to any given Number of Days, may be found by the Logarithms.

Thus, { *From the Log. of the given Number of Days, Subſtract the Log. of* 365 *Days,* (viz. 2.562293) *and there will remain the Log. of the Decimal parts equal to thoſe Number of Days.*

Example.

Let it be required to find the Decimal Parts of a Year equal to 135 Days; As before.

Here is given 135 its Logar. is 2.130334 } Subſtract
Days in a Year 365 its Log. is 2.562293 }

And there remains the Log. $\bar{9}$.568041 of 0,369863 the Decimal Parts equal to 135 Days, &c. as above.

Theſe things being underſtood, it will be as eaſie to Calculate any Queſtion relating to any of the foregoing Caſes, when the Time given, or ſought is either Leſs than a Year, or Years and Parts of a Year; as it is for thoſe in whole Years only.

As for Inſtance in *Caſe* 1. Suppoſe it were required to find *What Sum* 780 l. 15 s. *would Amount to in* 3 *Years and* 179 *Days, at Five and a Half* per Cent.?

In this Queſt. there is given *P* = 780,75 *R* = 0,055 and *T* = 3,4904 found by the Table.

Thus, Three Years = 3.000000 }
Days { 100 = 0,273973 } Add
70 = 0,191781 }
9 = 0,024657 }

Hence 3 Years and 179 Days is = 3,490411 = *T.*

Then *P* = 780,75 its Log. 2.892512 }
T = 3,4904 its Log. 9.542875 } Add
R = 0,055 its Log. 8.740363 }

The Sum is the Logarithm 2.175750 of 149,88.

That is, 149,88 = 149 *l.* 17 *s.* 7 *d.* the Inter. To which
Add the Principal 780 *l.* 15 *s.* 0 *d.*

And the Sum is 930 *l.* 12 *s.* 7 *d.* the *Amount* requir'd.

Again, That I may make all as plain as I can, take an *Example* in *Caſe* 4. *viz.* To find the Time.

Let it be requir'd to find, *In what Time* 780 l. 15 s. *would Amount to the Sum of* 930 l. 12 s. 7 d. *at the Rate of* 5½ per Cent. per Annum ?

In this Example there is given $A = 930{,}63$ $R = 0{,}055$ and $P = 780{,}75$ To find T: Which is done thus:

930,63—780,75=149,88 its Log. 2.175750
And $P = 780{,}75$ its Log. 2.892512 } Add
$R = 0{,}055$ its Log. $\bar{8}$.740363 }
From the firſt Log. Subſt. this Log. 1.632875
And there remains the Log. 0.542875 of 3.4904

That is, 3 Years, and 0,4904 Decimal Parts of a Year; but 0,4904 = 179 Days, *per* Rule in *pag.* 8. Conſequently 3 Years and 179 Days is the Time requir'd by the Queſtion.

Or the Decimal Parts of a Year may be otherwiſe Reduced into Days, by help of the laſt Table;

As in this Example; From 0,4904 the given Parts, Take the next leſs Tabular N°. 0,2740 = 100 Days.
There remains 0,2164
Again, From the Rem. take 0,1918 = 70 Days.
Laſtly this 0,0246 = 9 Days.

So that 0,4904 = 179 Days, by the Table, *&c.* as before.

Or the Decimal Parts of a Year may be Reduc'd into Days by the Logarithms;

Thus, { *To the Log. of the propoſed Parts, Add the Log. of* 365, *the Days in a Common Year, and the the Sum will be the Log. of the Days requir'd.*

For Inſtance, In the Laſt Example, wherein the given Decimal Parts are 0,4904 Log. $\bar{9}$.690550 } Add
The Days in a Year are 365 its Log. 2.562293 }
The Sum is this Logarithm 2.252843 of 179 *&c.*

Sect.

Sect. 2. Of **Annuities**, or **Penſions**, *&c.* in *Arrears*, Computed at *Simple Intereſt*.

I ſhall here make uſe of the ſame Letters to denote the ſeveral Parts of the Queſtion, as in the aforeſaid *Young Mathematician's Guide*, p. 248.

Viz.

U.	Denotes the Annuity or Penſion, *&c. viz.* either Yearly, Half Yearly, or Quarterly Rents.
T.	The Time of its being unpaid; *viz.* the Number of all the Payments that are in Arrears.
A.	The Amount of the Annuity and its Intereſt; *viz.* the Sum of all the Arrears due.
R.	The Ratio of the Rate, *viz.* the Intereſt of 1 *l.* As before.

Then will $$\frac{TTR - TR}{2} = \frac{A - TU}{U}.$$

From this Æquation is deduced the following Rules, which admits of Four Caſes.

Note. *I do here take it for granted, that the Reader doth by this time very well know how to perform Multiplication, and Diviſion of any given Numbers, by Logarithms; (as directed in* Chap. II. *and practiſed in all the Examples of the Laſt* Section: *) And therefore I ſhall, for Brevity ſake, omit ſetting down in Words at length (as in the laſt Section) how the following Caſes are to be Reſolv'd by Logarithms; that Work being ſo eaſily underſtood, by having a due regard to their reſpective Rules; That, it ſeems to me, to be wholly needleſs to repeat in Words how it is to be done; But only to ſet down the Work, at large of all the Examples, according to the Import of their reſpective Rules, by which they are Computed with the Pen only.*

Caſe 1. Having U, T, and R, given; To find A. That is, If any Annuity, or Rent, with the Time of its being Unpaid, and the Rate of Intereſt *per Cent.* be given; thence to find what Sum all thoſe Arrears will Amount to in that Time; allowing any aſſigned Rate of Simple Intereſt, for every particular Payment as it becomes due.

Rule. { *Multiply the Time, the Time leſs* 1, *and Half the Ratio of the Rate all three together, and to their Product Add the Time; Then Multiply that Sum with the Annuity, and the Product will ſhew the Amount requir'd.*

Example.

Suppoſe 356 *l.* Annuity, or Yearly Rent, be forborn or unpaid Nine Years; *What Sum will all thoſe Arrears Amount to in that Time, allowing* 6 per Cent. per Ann. *for each Payment as it becomes due?*

In this Example there is given, $U = 356$; $T = 9$; and $R = 0{,}06$; to find A.

Firſt by the *Pen* only.

Let the Work be prepared thus, $9 - 1 = 8$, And $\frac{1}{2} R = 0{,}03$

Then $9 \times 8 = 72$; and $72 \times 0{,}03 = 2{,}16$; To which Adding the Time, it will be $2{,}16 + 9 = 11{,}16$. And 11,16 Multiplied with 356 is $3972{,}96 = A$.

That is 3972 *l.* 19 *s.* 2½ *d.* will be the Amount as was requir'd.

The ſame perform'd by *Logarithms*.

Firſt $T = 9$ its Logar. is 0.954242
And $T - 1 = 8$ its Logar. is 0.903090 } Add
Again, $\frac{1}{2} R = 0{,}03$ its Log. is $\bar{8}.477121$
Their Sum is the Log. 0.334453 of 2,16

Then $2{,}16 + 9 = 11{,}16$ its Log. is 1.047664 } Add
And $U = 356$ its Log. is 2.551450
The Sum is the Logarithm 3.599114 of 3972,96

That is 3972 *l.* 19 *s.* 2½ *d.* $= A$, the Amount or Sum of all the Arrears, as before by the Pen.

Caſe 2. When *A*, *T*, and *R*, are given, to find *U*. That is, To find what Annuity, or Yearly Rent, being forborn or unpaid any aſſign'd Time, will Amount to a propoſed Sum, allowing any given Rate of Intereſt *per Cent.* for every Payment as it becomes due.

Rule. *Multiply the Time, the Time leſs* 1, *and Half the Ratio of the Rate, all three together; and to their Product, Add the Time (as before) Then Divide the propoſed Amount by that Sum, and the Quotient will ſhew the Annuity, or Yearly Rent requir'd.*

Example.

Suppoſe it were requir'd to find, *What Annuity, or Yearly Rent will Amount to (or Raiſe a Stock) of* 3972 l. 19 s. 2½ d. *allowing* 6 per Cent. *for every Payment as it becomes due?*

In this Queſtion there is given $A = 3972{,}96$; $T = 9$; and $R = 0{,}06$ To find U; which may be thus done.

First by the *Pen* only; Thus,

The Work prepared is $9 - 1 = 8$, and $\frac{1}{2} R = 0{,}03$ Then $9 \times 8 = 72$, and $72 \times 0{,}03 = 2{,}16$; which being Added to the Time, is $9 + 2{,}16 = 11{,}16$ for a *Diviſor.*

Then $11{,}16 \,)\, 3972{,}96 \,(\, 356 = U$

Viz. 356 *l.* will be the Yearly Rent requir'd.

The ſame perform'd by *Logarithms.*

Thus, $T = 9$ its Log. is 0.954242 }
$T - 1 = 8$ its Log. is 0.903090 } Add
And $\frac{1}{2} R = 0{,}03$ its Log. is 8.477121 }
The Sum is the Logarithm 0.334453 of 2,16

Again, $A = 3972{,}96$ its Log. is 3.599113 }
And $9 + 2{,}16 = 11{,}16$ its Log. 1.047664 } Subſtact
There remains the Logarithm 2.551449 of $356 = U$

That is, 356 *l.* is the Annuity, or Yearly Rent, As before.

Caſe 3. When *U*, *A*, and *R*, are given; To find *T*. That is, To find the *Time* in which any Annuity, or Yearly Rent, being forborn or unpaid, will Amount to any propoſed Sum; allowing any given Rate of Intereſt *per Cent.* &c. for each Payment as it becomes due.

Rule. *Subſtract the Ratio of the Rate from* 2; *then Divide the Remainder by twice the Ratio, and call the Quotient* x.

Next, Square that Quotient (viz. Multiply it with it ſelf) *and call that Square* xx.

Then Divide Twice the propoſed Amount by the Product of the Annuity Multiply'd with the Ratio, and to the Quotient, Add the Square Number called xx: *Then Extract the Square Root of that Sum, and from that Root Subſtract the Number called* x, *and the Remainder will ſhew the Time ſought.*

Example.

What Time will 356 l. *Yearly Rent require to Raiſe the Sum of* 3972 l. 19 s. 2½ d. *at Six* per Cent. &c. *for each Payment as it becomes due?*

In this Queſtion there is given, $U = 356$; $R = 0{,}06$; and $A = 3972{,}96$; To find *T*.

And firſt by the *Pen* only.

1ſt $2 - 0{,}06 = 1{,}94$ And $2R = 0{,}12$) 1,94 ($16{,}166 = x$.

Next, $16{,}166 \times 16{,}166 = 261{,}361$ the Num. call'd *xx*.

Then $2A = 7945{,}92$ and $356 \times 0{,}06 = 21{,}36$
And 21,36) 7945,92 (372,003

Then $372{,}003 + 261{,}361 = 633{,}364$ whoſe Square Root is 25,166; Laſtly, $25{,}166 - 16{,}166$, Leaves $9 = T$; *Viz.* 9 Years, is the Time requir'd by the Queſtion.

The ſame found by *Logarithms.*

Firſt 2 — 0,06 = 1,94 its Log. 0.287802 } Subſtract
And 2 *R* = 0,12 its Logar. is $\bar{9}$.079181 }

There Remains the Logar. 1.208621 of 16,166
Multiplied with 2 (call'd *x*.

The Product is the Logarithm 2.417242 of 261,361
(call'd *xx*.

Again; 2 *A* = 7945,92 its Log. 3.900145

And *U* = 356 its Log. is 2.551449 } Add
R = 0,06 its Log. is $\bar{8}$.778151 }

From the 1ſt Log. Sub. this Log. 1.329600

And there remains the Logar. 2.570545 of 372,003

Then 372,003 + 261,361 = 633,264 its Log. 2.801585
The Half of that Log. is 1.400792

Laſtly, the Number which belongs to the Half Logarithm is 25,166 From which Subſt. the Number called *x* = 16,166

Remains 9,000 = *T*; the Num. of Years ſought, &c.

Caſe 4. *If U, T,* and *A,* are given; To find *R.* That is, Having any Annuity, or Yearly Rent, with the Time of its being Unpaid, and the Sum it's propoſed to Amount to in that Time given; Thence to find, what Rate of Intereſt *per Cent.* muſt be allow'd for every Payment as it becomes due.

Rule. { *Multiply the Annuity with the Time, and Multiply that Product with the Time again; And make half the Difference of thoſe Two Products a Diviſor.*

Next, Subſtract the firſt Product from the propoſed Amount, then Divide the Remainder by the aforeſaid Diviſor, and the Quotient will be the Ratio of the Rate of Intereſt required.

Exam-

Example.

Suppoſe 356 l. *Yearly Rent, being forborn or unpaid Nine Years, be propoſed to Raiſe the Sum of* 3972 l. 19 s. 2$\frac{1}{2}$ d. *What Rate of Intereſt* per Cent. *muſt be allow'd for every Payment as it becomes due ?*

In this Queſtion there is given $U = 356$, $T = 9$; And $A = 3972{,}96$ To find R. Which may be thus found.

First by the *Pen* only.

Thus, $356 \times 9 = 3204$; And $3204 \times 9 = 28836$
Next, $28836 - 3204 = 25632$ its Half is 12816 for the Diviſor.

And $3972{,}96 - 3204 = 768{,}96$ for the Dividend.

Then 12816) 768,96 ($0{,}06 = R$ the Ratio.

And it will be, As 1 : Is to 0,06 : : So is 100 : To 6 the Rate of Intereſt *per Cent.* &c. As was required.

The ſame may be done by *Logarithms.*

$$\begin{array}{lr} \text{Thus } U = 356 \text{ its Log.} & 2.551450 \\ \text{And } T = 9 \text{ its Log.} & 0.954242 \end{array}\Bigg\}\ \text{Add}$$

To the laſt Log. Add this 3.505692 its Num. is 3204
Their Sum 4.459934 its Num. is 28836
The Difference of the Two Numbers is 25632
and the $\frac{1}{2}$ of 25632 is 12816 for the Diviſor.

From $A = 3972{,}96$ take the 1ſt Num. 3204, and

$$\begin{array}{lr} \text{there remains } 768{,}96 \text{ its Log.} & 2.885904 \\ \text{and } 12816 \text{ the Diviſ. its Log.} & 4.107752 \end{array}\Bigg\}\ \text{Subſtract}$$

There Remains the Logar. $\bar{8}.778152$ of $0{,}06 = R$

Then 1 : 0,06 : : 100 : 6 the Rate of Intereſt *per Cent.* &c.

Thus you have all the Four *Caſes* relating to *Annuities*, or *Rents*, &c. in Arrears, with their Examples in Yearly Payments: But if the *Annuities*, or *Rents*, are to be paid by Half-yearly, or Quarterly Payments, as moſt generally they are,

Then,

1. Inſtead of the Ratio of the given Rate of Intereſt, you muſt take the $\frac{1}{2}$ of that Ratio for Half-yearly Payments, and the $\frac{1}{4}$ of it for Quarterly Payments, *&c.*

2. And you muſt take the $\frac{1}{2}$ of the Yearly Rent for Half-yearly Payments, and the $\frac{1}{4}$ of it for Quarterly Payments, *&c.*

3. But inſtead of the propoſed Number of Years, you muſt take Twice that Number for Half-yearly Payments, and Four times that Number for Quarterly Payments, *&c.* As in the following Examples.

Examples in Half-yearly Payments.

Suppoſe 356 l. per Annum *Annuity, payable every Half-year, were forborn or unpaid Nine Years; What would all thoſe Arrears Amount to, at the Rate of* 6 per Cent. per Annum, &c.?

In this Queſtion there is given $U = 178$, *viz.* the $\frac{1}{2}$ of 356 *l.* $R = 0{,}03$ the $\frac{1}{2}$ of the Ratio of 6 *per Cent.* and $T = 18$, *viz.* 9×2 the Number of Half-years in Nine Years; Thence to find *A.* (*per Rule* at *Caſe* 1.)

Firſt to prepare the Work $18 - 1 = 17$; and $\frac{1}{2} R = 0{,}015$.

Then $18 \times 17 = 306$; and $306 \times 0{,}015 = 4{,}59$ To which Add the Time, *viz.* $4{,}59 + 18 = 22{,}59$ And then $22{,}95 \times 178$ gives $4021{,}02 = A$, the Amount or Sum of all the Arrears, as was requir'd.

The ſame Example in Quarterly Payments.

That is, 356 l. *a Year, to be paid every Quarter, being forborn Nine Years, What Sum will it Amount to, allowing* 6 per Cent. &c. *for every Payment as it becomes due?*

Now here it will be $U = 89$, *viz.* the $\frac{1}{4}$ of 356 *l.* $R = 0{,}015$, *viz.* the $\frac{1}{4}$ of 0,06 the Ratio of the given Rate *per Cent.* &c. And $T = 36$, *viz.* 9×4 the Number of Quarters in Nine Years: Thence to find *A*, as before.

To Prepare the the Work $36 - 1 = 35$, and $\frac{1}{2} R = 0{,}0075$.

Then $36 \times 35 = 1260$; and $1260 \times 0{,}0075 = 9{,}45$ to which Add the Time, *viz.* $9{,}45 + 36 = 45{,}45$ and then $45{,}45 \times 89$ gives $4045{,}05 = A$, the Amount or Sum of all the Arrears at Quarterly Payments.

By comparing the Work of theſe Two Examples with that in Page 33, it may be obſerv'd, That Half-yearly Payments are more advantagious than Yearly Payments; and Quarterly are yet more advantagious than Half-yearly:

For Yearly Payments Amount but to 3972 *l.* 19 *s.* $2\frac{1}{2}$ *d.*
But $\frac{1}{2}$ yearly Payments Amounts to 4021 *l.* 00 *s.* $3\frac{1}{2}$ *d.*
And Quarterly Paym. Amounts to 4045 *l.* 01 *s.* 00 *d.*

If the two laſt Examples be well conſider'd and underſtood, it will be eaſie to conceive how to Compute any Queſtion in the other Three Caſes, when the Payments are either Half-yearly, or Quarterly; and therefore I ſhall omit inſerting Examples, and proceed to the next Section.

Sect. 3. The Preſent Worth of Annuities and Penſions, &c. Computed at Simple Intereſt.

In this *Section* I ſhall make uſe of theſe Letters to denote the ſeveral Parts of the Queſtion:

Viz. *U.* Denotes the *Annuity* or *Rent.* As before.
T. The *Time* of its *Continuance.*
R. The *Ratio* of the *Rate* of *Intereſt.*
And *P.* Denotes the *Preſent Worth* of the *Annuity.*

Then

Then $TTRU - TRU + 2TU = 2P + 2TRP$ (*Vide* the aforeſaid *Young Mathematician's Guide*, p.251.) And from this Æquation are deduced the following Rules, which alſo admits of Four Caſes, as in the laſt Section.

Caſe 1. Having *U*, *T*, and *R*, given; To find *P*.

That is, Having any Annuity or Yearly Rent, and the Time of its Continuance given; To find the Preſent Worth of that Annuity, or Leaſe, &c. allowing any given Rate of Simple Intereſt *per Cent.* to the Purchaſer for his ready Money.

Rule. *Multiply the Time, the Time leſs* 1, *and Half the Ratio of the Rate, all three together, and to their Product Add the Time; Then Multiply that Sum with the Annuity.* (So far the Work is juſt the ſame as in the laſt *Section.*) *Then Divide that Product, by the Product of the Ratio into the Time, Added to* 1. *and the Quotient will ſhew the Preſent Worth requir'd.*

Example.

Suppoſe a Leaſe of 250 l. per Annum, *the Rents to be paid Half-yearly, were to be Lett for* 21 *Years; What may the preſent Worth of that Leaſe be, at the Rate of* 5 per Cent. &c.

In this Queſtion there is given $U = 125$, *viz.* the $\frac{1}{2}$ of 250; $R = 0,025$ the $\frac{1}{2}$ of 0,05 the Ratio of 5 *per Cent.*; and $T = 42$, (*viz.* 21 × 2) the Number of Half-Years in 21 Years; To find *P*.

Firſt, the Work Prepared ſtands thus, $42 - 1 = 41$, and $\frac{1}{2} R = 0,0125$.

Then $42 \times 41 = 1722$; and $1722 \times 0,0125 = 21,525$ Next, $21,525 + 42 = 63,525$ which Multiplied with 125, the Half-yearly Payment, is 7940,625 for the Dividend.

Again, $42 \times 0{,}025 = 1{,}05$, and $1{,}05 + 1 = 2{,}05$ r the Diviſor.

M Then $2{,}05\,)\,7940{,}625\,(\,3873{,}4756 = P$.

t. 3873 *l.* 9 *s.* 6 *d.* will be the Preſent Worth of h a Leaſe as was requir'd.

The ſame perform'd by *Logarithms.*

Firſt $T = 42$ its Log. is 1.623249
And $T - 1 = 41$ its Log. is 1.612784 } Add
$\frac{1}{2}R = 0{,}0125$ its Log. is $\bar{8}.096910$

The Sum is the Logarith. 1.332943 of 21,525
And $21{,}525 + 42 = 63{,}525$ its Log. 1.802945 } Add
$U = 125$ its Log. is 2.096910

The Sum is the Log. of the Dividend 3.899855

Again, $T = 42$ its Logar. is 1.623249 } Add
And $R = 0{,}025$ its Log. is $\bar{8}.397940$

The Sum is the Logarithm 0.021189 of 1,05;
And $1{,}05 + 1 = 2{,}05$ for the Diviſor.

Then the Log. of the Divid. is 3.899855 } Subſtract
And the Log. of 2,05 the Diviſ. 0.311754

There Remains the Log. 3.588101 of 3873,47

That is, 3873 *l.* 9 *s*, 4¼ *d.* is here found to be the Preſent Worth of that Annuity; As before, *Ferè*.

Caſe 2. When P, T, and R, are given; to find U. *Viz.* To find what Annuity, or Leaſe, &c. may be Purchaſed for any propoſed *Sum*, to continue any aſſigned Time, and at any given Rate of Intereſt.

Rule. *Multiply the Time, the Ratio of the Rate, and the preſent Worth,* (viz. the Purchaſe-Money) *all three together; and to their Product Add the preſent Worth; make that Sum a Dividend: Then Multiply the Time, the Time leſs* 1, *and half the Ratio together; and to their Product Add the Time; make that Sum a Diviſor; the Quotient ariſing from thence, will ſhew the Annuity, or Rent required.*

Exam-

Example.

What Annuity, or Rent, to be paid Half-yearly, and to continue 21 Years, may be Purchaſed for the Sum of 3873 l. 9 s. 6 d. *at the Rate of* 5 per Cent.?

In this Example there is given $P = 3873,475$ $T = 42$ *viz.* 21×2, the Half-Years in 21 Years, and $R = 0,25$ the $\frac{1}{2}$ of the Ratio 0,05 of 5 *per Cent.*; To find U, the Half-yearly Rent.

The Work Prepared is $42 - 1 = 41$; and $\frac{1}{2} R = 0,0125$

Then $3873,475 \times 42 = 162685,95$ and $162685,95 \times 0,025$ gives $4067,14875$; And $4067,14875 + 3873,475 = 7940,62375$ for the Dividend.

Again, $42 \times 41 = 1722$ and $1722 \times 0,0125 = 21,525$ and $21,525 + 42 = 63,525$ for the Diviſor.

Then $63,525\ )\ 7940,62375\ (\ 125 = U$, the Half-yearly Rent, conſequently $125 \times 2 = 350$ *l.* will be the Annuity or Rent *per Annum*, as was required.

The ſame done by *Logarithms.*

First $P = 3873,475$ its Log. is 3.588101
And $T = 42$ its Log. is 1.623249 } Add.
$R = 0,0125$ its Log. is 8.397940

The Sum is the Logarithm 3.609290 of 4067,15

And $4067,15 + 3873,475 = 7940,625$ the Dividend.

Again, $T = 42$ its Log. is 1.623249
$T - 1 = 41$ its Log. is 1.612784 } Add.
And $\frac{1}{2} R = 0,0125$ its Log. is 8.096910

The Sum is the Logarithm 1.332943 of 21,525

And $21,525 + 42 = 63,525$ for the Diviſor.

Then the Divid. 7940,625 its Log. 3.899854 } Subſt.
The Diviſor 63,525 its Log. is 1.802945

There remains the Logar. 2.096909 of 125 *Viz.* 125 *l.* is the Half-yearly Rent, *&c.* As before.

Caſe 3. When *U*, *P*, and *R*, are given; To find *T*. That is, when any Annuity, with its propoſed Value or Purchaſe, and the Rate of Intereſt *per Cent.* are given; Thence to find how Long the Purchaſer ought to enjoy it.

Rule. *To the preſent Worth, Add half the Annuity; then Multiply that Sum with the Ratio, and Subſtract the Product from the Annuity; Then Divide the Remainder by the Product of the Annuity Multiplied with the Ratio, and call the Quotient x.*

Next, Square that Quotient (viz. *Multiply it with itſelf*) *and call that Square xx.*

Then Divide the preſent Worth by the ſame Diviſor, viz. *by the Product of the Annuity into the Ratio; and to twice that Quotient Add the Square Number called xx; Then Extract the Square Root of their Sum, and from that Root Subſtract the Number called x; The Remainder will ſhew the Time ſought.*

Example.

Suppoſe one would Lay out 3873 l. 9 s. 6 d. *ready Money for an Annuity of* 250 l. per Annum, *to be paid by Half-yearly Payments; How long may he enjoy that Annuity, to be allow'd but* 5 per Cent, per Annum, *Simple Intereſt for his Money.*

Or thus, which is the ſame thing:

In what Time will 250 l. per Annum, *Pay off a Debt of* 3873 l. 9 s. 6 d. *by Half-yearly Payments, allowing the Creditor* 5 per Cent. *Intereſt for his Money, until the Debt be Diſcharged?*

In this Queſtion there is given $P = 3873,475$ $U = 125$, *viz.* the Half-yearly Payments, and $R = 0,025$ *viz* the $\frac{1}{2}$ of the Ratio of 5 *per Cent.* Thence to find T, the Number of Half-Years.

Firſt $\frac{1}{2}U = 62,5$ and $62,5 + 3873,475 = 3935,975$

Then 3935,975 × 0,025 = 98,4 *ferè*, and 125 − 98,4 Leaves 26,6 for the firſt Dividend.

And 125 × 0,025 = 3,125 for the Common Diviſor.

Then 3,125) 26,600 (8,512 the Number call'd x.

And 8,512 × 8,512 = 72,454 the Number call'd xx.

Again, the Common Diviſor is 3,125

And 3,125) 3873,475 (1239,512

$$\left.\begin{array}{lr}\text{And twice } 1239{,}512 \text{ is} & 2479{,}024\\ \text{The Number call'd } xx \text{ is} & \underline{72{,}454}\end{array}\right\}\text{Add}$$

And the Square Root of 2551,478 is 50,512

Laſtly 50,512 − 8,512 (*the Number call'd x*) leaves 42 = *T* the Number of Half-Years; conſequently the Half of 42, *viz.* 21 will be the Time or Number of Years required by the Queſtion.

This Caſe may alſo be Reſolved by Logarithms, but it is attended with ſome ſeeming Difficulties, which requires a due Conſideration of the Rule.

As for Inſtance in the Laſt Example, wherein *R* = 3873,475; *U* = 125; and *R* = 0,025; To find *T.*

$$\left.\begin{array}{rr}62{,}5 + 3873{,}475 = 3935{,}975 \text{ its Log.} & 3.595053\\ R = 0{,}025 \text{ its Log. is} & \underline{8.397940}\end{array}\right\}\text{Add}$$

The Sum is the Log. of 98,4 —— 1.992993

125 − 98,4 = 26,6 its Log. 1.424882 of the *Divid.*

$$\left.\begin{array}{lr}\text{Again, } U = 125 \text{ its Log. is} & 2.096910\\ \text{And } R = 0{,}025 \text{ its Log. is} & \underline{8.397940}\end{array}\right\}\text{Add}$$

The Sum is the Log. of 3,125 : 0.494850 the *Diviſor*.

To be Subſt. from the upper Log. 0.930032 Remains the Logarithm of 8,512 the Number called x.

And the Double of the laſt Log. is 1.860064 which is the Logarithm of 72,454 the Number call'd xx.

Again, $P=3873,475$ its Log. 3.588100 } Subſtract
And the Log. of the Diviſor is 0,494850 }

There remains the Logarithm 3.093250 of 1239,512

Twice 1239,512 is 2479,024 } Add
The Num. call'd xx 72,454 }

The Sum is 2551,478 its Log. is 3.406792

The Half of the laſt Log. is 1.703396 its Number is 50,512 from which Subſtract the Numb. call'd $x=$ 8,512

Remains 42,000 $=T$ the Number of Half-Years, &c. As before.

Caſe 4. When *U*, *P*, and *T*, are given; To find *R*.

That is, when any Annuity, with the Time of its Continuance, and its propoſed Preſent Worth for that Time are given; Thence to find what Rate of Intereſt *per Cent.* is allow'd the Purchaſer.

Rule. *Multiply the Annuity with the Time, and from the Product Subſtract the preſent Worth; then make twice that Remainder a Dividend.*

Next, Add the Annuity to twice the preſent Worth, and from their Sum Subſtract the firſt Product; then Multiply the Remainder with the Time, and make the Product a Diviſor; by which Divide the aforeſaid Dividend, and the Quotient will ſhew the Ratio of the Rate of Intereſt requir'd.

Example.

Suppoſe 3873 L 9 s. 6 d. *were given for an Annuity of* 250 l. per Annum; *to be paid Half-yearly, and to continue* 21 *Years; What Rate of Intereſt* per Cent. &c. *is allow'd the Purchaſer.*

Here is given $P=3873,475$; $U=125$ the Half-yearly Payments; and $T=42$, the Number of Half-Years in 21 Years.

Then $125 \times 42 = 5250$ the firſt Product;
And $5250 - 3873{,}475 = 1376{,}525$ its Double is $2753{,}05$ for the Dividend.

Next, $2P = 7746{,}95$ and $7746{,}95 + 125 = 7871{,}95$ and $7871{,}95 - 5250$ (*the firſt Product*) leaves $2621{,}95$

Again, $2621{,}95 \times 42 = 110121{,}9$ for a Diviſor.
Then $110121{,}9\,)\,2753{,}05\,(\,0{,}025 = R$ the Ratio of the Rate *per Cent.* for Half-yearly Payments, conſequently $0{,}025 \times 2 = 0{,}05$ is the Ratio of the Rate *per Cent, per Annum*; And then it will be,
As 1 : To 0,05 :: So is 100 : To 5, the Rate of Intereſt as was requir'd.

The ſame may be done by *Logarithms*.

Thus, $U = 125$ its Log. is 2.096910 } Add
And $T = 42$ its Logar. is 1.623249 }
The Sum is the Logarithm 3.720159 of 5250

And $5250 - 3873{,}475 = 1376{,}525$ the Double whereof is 2753,05 its Log. is 3.439814 *to be Reſerv'd.*
Next $2P = 7746{,}95$ and $7746{,}95 + 125 = 7871{,}95$ and $7871{,}95 - 5250$ the firſt Product or Num. above,
There remains 2621,95 its Log. is 3.418624 } Add
The Time $42 = T$, its Logar. is 1.623249 }
Subſtract this Logarithm 5.041873 Sum
From the Reſerved Logar. *viz.* 3.439814
And there remains the Logarith. 8.397941 of 0,025
Viz. $0{,}025 = R$ the Ratio of the Rate *per Cent.* for Half-yearly Payments, *&c.* As above.

Theſe Four Examples may be ſufficient to ſhew how any other of the like kind may be Reſolved, either for whole Years, or thoſe of Quarterly Rents, *&c.* provided the Purchaſed Rent, Leaſe, or Annuity is to commence immediately.

But if it be required to find the preſent Value, or Worth of any Rent, or Annuity, *&c.* in Reverſion;

That is, when it is not to be Enter'd upon until after ſome Time or Number of Years are paſt.

Then { *You muſt firſt find what the propoſed Annuity, or Rent,* &c. *would be worth for the given Time of its Continuance, as if it were to be immediately Enter'd upon:*

And then you muſt find what Principal or Sum being forborn at Intereſt during the Time of that Reverſion, would Amount to or Raiſe the aforeſaid Value; That Principal will be the Sum which ſhould be paid for the propoſed Annuity in Reverſion.

Example.

There is the Reverſion of a Leaſe of 175 l. per Annum, *to be Lett for Eleven Years, which are to commence after Nine Years are expired; 'Tis required to find the preſent Worth of that Leaſe, allowing the Purchaſer* 6 per Cent. *for his ready Money.*

The firſt Work in this Queſtion muſt be to find what 175 *l. per Annum*, to continue 11 Years, is worth in ready Money, ſuppoſing it were to be immediately Enter'd upon: As in *Caſe* 1. Page 40.

That is, here is given $U = 175$; $T = 11$; and $R = 0{,}06$ To find P, the Preſent Worth.

The Work Prepared will ſtand thus $11 - 1 = 10$ and $\frac{1}{2} R = 0{,}03$. Then $11 \times 10 \times 0{,}03 = 3{,}3$ And $3{,}3 + 11 = 14{,}3$.

Next, $14{,}3 \times 175 = 2502{,}5$ for a Dividend.

Again, $11 \times 0{,}06 = 0{,}66$ and $0{,}66 + 1 = 1{,}66$ the Diviſ.

Then 1,66) 2502,5 (1507,53 $= P$ the Preſent Worth, if the Leaſe were to commence immediately, but becauſe it is not ſo, Therefore

The next Work muſt be to find what Principal or Sum, being put out to Intereſt for Nine Years, at

6 *per Cent.* &c. will Amount to 1507,53 *l.* the which will be found (*per Caſe* 2. *p.* 24.)
To be 978,9162 = 978 *l.* 18 *s.* 3¼ *d.* which is the true preſent Worth of that Leaſe in Reverſion, as was required.

2. Or if it be required to find what Annuity, &c. in Reverſion, may be Purchaſed for any propoſed Sum, and at any given Rate of Intereſt *per Cent.* when the Time the Annuity is not to be Entered upon, and the Time of its Continuance are both given.

Then

The firſt Work muſt be to find what the Sum propoſed to be laid out in the Reverſion, would Amount to in that Time wherein the Annuity is not to be in Poſſeſſion, as if it were forborn at Intereſt during that Time, at any given Rate per Cent. (by Rule 1. Page 23.)

And the ſecond Work will be to find what Annuity, &c. *that Amount will purchaſe*, (as in Caſe 2. Page 41.) *and that will be the Anſwer to the Queſtion.*

Theſe Two are the moſt general and uſeful Queſtions that relate to Purchaſing Annuities, or Leaſes in Reverſion: Not but that, if there be occaſion, either the Time, or the Rate of Intereſt may be found by a due Application of their reſpective Rules.

I know that Sir *Samuel Moreland*, and ſeveral other Authors, that Treat upon the Subject of Intereſt and Annuities, do aſſert, That neither the Arrears, nor the preſent Worths of Annuities can be truly Computed at Simple Intereſt, but only at Compound Intereſt; And therefore, ſay they, all Computations of Annuities, or Leaſes, &c. at Simple Intereſt are to be laid aſide as uſeleſs.

'Tis true indeed, that in Purchaſing of Annuities, or Taking of Leaſes, either to be in preſent Poſſeſſion, or in Reverſion; it is uſual to allow the Purchaſer Compound Intereſt for his ready Money, notwithſtanding it be not lawful to Lett out Money at Intereſt upon Intereſt, *viz.* at Compound Intereſt.

However, that we may the better judge whether ſuch Computations as are made of Annuities in Arrears at Simple Intereſt, be uſeful or no: Let us here ſee and compare how near they will agree with the Ninety Nine Years Annuities, ſettled by the late Acts of Parliament.

Thoſe Annuities were Sold at Sixteen Years Value, or Purchaſe; That is, 1600 *l.* ready Money Bought 100 *l.* *per Annum*, to continue 99 Years, and ſo in proportion for a Greater, or Leſſer Sum.

Now it may be reaſonable to ſuppoſe, that, the Parliament deſign'd at the Settling of thoſe Annuities at that Value, to allow the Purchaſer after the Rate of Six *per Cent. per Annum* Simple Intereſt for his ready Money, according to the former Laws relating to Intereſt: If ſo, the Intereſt of 1600 *l.* would be 96 *l.* *per Annum* for ever, (*as by Caſe* 1. *Pag.* 22) conſequently there will remain but 4 *l.* *per Annum* for 99 Years in Lieu of the Principal or Purchaſe-Money: That is, there will be but $99 \times 4\,l. = 396\,l.$ repaid inſtead of the 1600 *l.* If it be only receiv'd without any further Conſideration.

But, if we Compute what that 4 *l.* *per Annum* would Amount to, if it were forborn in Arrears for 99 Years, allowing 6 *per Cent.* for every Payment as it becomes due, &c. (*per Caſe* 1. *Pag.* 33) Then will that Amount undoubtedly ſhew what Sum will be truly repaid for the 1600 *l.* firſt paid for that Annuity.

And in order to that there is given $U = 4$ $T = 99$ and $R = 0{,}06$ the Ratio of the Rate of 6 *per Cent.* To find A, the Amount.

Which Prepared for the Work is $99 - 1 = 98$
And $\frac{1}{2} R = 0{,}03$

Then $99 \times 98 = 9702$, and $9702 \times 0{,}03 = 291{,}06$
To find A, the Amount.

Next $291{,}06+99=390{,}06$, and $390{,}06\times4=1560{,}24$ equal *A*, *viz.* 1560 *l.* 4 *s.* $9\frac{1}{2}$ *d.* is the Sum that will in effect be received for the 1600 *l.* over and above the 96 *l. per Annum* Intereſt.

Now in this Calculation I take for granted what I have elſewhere proved; (*Vide Young Mathematician's Guide, Page* 248) *viz.* That if any Yearly Rent be forborn, or Let run in Arrears, and any Rate of Simple Intereſt be allow'd for every Payment as it becomes due; the Amount, or Sum of all thoſe Payments will be the very ſame, as if every Year's Rent were actually receiv'd, and immediately put out to Intereſt at the ſame Rate.

Whence it follows, That altho' 396 *l.* be all the Money that's actually received in the ſpace of 99 Years, over, or above the 96 *l. per Annum* Intereſt; yet according to the Nature of Intereſt, there is effectually 1560 *l.* 4 *s.* $9\frac{1}{2}$ *d.* receiv'd; ſuppoſing the Annuity to be paid but Yearly, which is indeed paid Quarterly; and therefore the Sum will be ſtill nearer to 1600 *l.* which plainly ſhews that the Computations of Annuities, or Rents run in Arrears at Simple Intereſt, are not to be wholly rejected as uſeleſs.

But I cannot pretend to ſay much in the behalf of ſuch Computations as are made about the preſent Worth of Annuities, or Leaſes, &c. at Simple Intereſt, nor are they to be relied on in Practice: However, becauſe this Tract ſhould not be wanting in that Part, I thought it convenient to ſhew (*altho' but briefly*) how thoſe Computations may be perform'd.

And ſhall in the next Chapter handle the Buſineſs of Purchaſing Annuities, &c. more fully; eſpecially that Part of it which relates to Annuities, or Leaſes in Reverſion.

CHAP.

CHAP. IV.

The *Calculation* of *Queſtions* in Compound Intereſt, and Annuities; perform'd by the help of *Logarithms*.

Compound Intereſt is that which is produced from any Principal and its Intereſt put together, as the Intereſt of that Principal becomes due: That is, at every Payment, Or rather at the Times when the Payments become due, there is ſtill created a New Principal, by the Increaſe of the growing Intereſt; and therefore it is called *Intereſt upon Intereſt*, or *Compound Intereſt*; which is grounded upon a Series of Geometrical Proportionals continued, as I have plainly demonſtrated in the *Young Mathematician's Guide*, P. 253, &c.

In all Computations of this kind, whether they are about Money forborn at Intereſt, or thoſe relating to Annuities, &c. we generally make uſe of the Amount or Produce of One Pound and its Intereſt for One Year, inſtead of the Ratio of the given Rate of Intereſt; As before in Simple Intereſt.

And that Amount of 1 *l.* is no more but the Ratio of the given Rate Added to a Unit or 1.

For the Ratio is the Intereſt of 1 *l.* (*vid. Pag.* 20) conſequently if that 1 *l.* be Added to it, the Sum will be the Amount of 1 *l.* (*per Rule at Caſe* 1. *Pag.* 23) Or

The Amount of 1 *l.* for One Year at any given Rate of Intereſt *per Cent.* may be ſound by this Proportion;

Viz.

As 100 : 106 :: 1 : 1,06 the *Amount* of 1 *l.* at 6 *per Cent.*
Or 100 : 107 :: 1 : 1,07 the *Amount* of 1 *l.* at 7 *per Cent.*
And ſo on for any other given Rate of Intereſt.

Or the aforeſaid *Amounts* of 1 *l.* may be otherways ſound by *Logarithms*;

Thus, { *Add the propoſed Rate of Intereſt to* 100, *and from the Logarithm of that Sum, Subſtract the Logarithm of* 100 (viz. 2.000000) *the Remainder will be the Logarithm of the Amount of* 1 l. *at that Rate of Intereſt* per Cent.

Example.

Suppoſe the given Rate of Intereſt to be that of 6 per Cent. per Annum.

Then 100 + 6 = 106 its Log. is 2.025306 } Subſt.
And the Log. of 100 is 2.000000 }

There Remains the Logarithm 0.025306 of 1,06 *Viz.* 1,06 is the Amount of 1 *l.* at 6 *per Cent.* &c. And ſo for any other Rate of Intereſt.

And becauſe 'tis the Logarithms of thoſe Amounts of 1 *l.* that are of greateſt Uſe in the following Calculations, I have here annexed a ſmall Table of ſeveral Rates of Intereſt, with the Amounts of 1 *l.* *per Annum* at thoſe Rates, and the Logarithms of thoſe Amounts.

Rates of Intereſt perCent. l.	*The Amounts of* 1 l. Viz. R.	*Logarithms of thoſe Amounts,* Viz. *of* R.	*Rates of Intereſt perCent.* l.	*The Amounts of* 1 l. Viz. R.	*Logarithms of thoſe Amounts,* Viz. *of* R.
3	1,03	0.012837	7	1,07	0.029384
4	1,04	0.017033	7½	1,075	0.031408
4½	1,045	0.019116	8	1,08	0.033424
5	1,05	0.021189	8½	1,085	0.035430
5½	1,055	0.023252	9	1,09	0.037426
6	1,06	0.025306	9½	1,095	0.039414
6½	1,065	0.027350	10	1,1	0.041393

Theſe things being underſtood, you may proceed to the Reſolving of Queſtions about Money forborn at Compound Intereſt.

Sect. 1. Of **Compound Intereſt**, Or *Money* Lett out at *Intereſt* upon *Intereſt*.

I ſhall here make uſe of the ſame Letters to denote the ſeveral Parts of the Queſtion, as before in *Pag.* 22. except *R* only.

Viz.
- *P*. Denotes any Principal put to Intereſt.
- *t*. The Time of its Continuance at Intereſt.
- *A*. The Amount or Principal and its Intereſt.

And *R*. Denotes the Amount of 1 *l.* for One Year. That is, the Ratio of the given Rate more 1.

Then it will be, $PR^t = A$.

This Æquation admits of the ſame Four Caſes, or Variety of Queſtions, as that of Simple Intereſt in *Page* 22. And here it may not be amiſs to make uſe of the ſame Examples, that we may the more readily ſee the Difference that is between Simple Intereſt, and Compound Intereſt.

Caſe 1. If *P*, *t*, and *R*, are given; To find *A* the Amount.

Rule. *Firſt Multiply the Log. of R, the Amount of* 1 l. *at the given* Rate per Cent. *with the Time, Then to that Product Add the Log. of the propoſed Principal, and their Sum will be the Log. of the Amount required.*

Example.

What Sum will 567 l. 10 s. *Amount to in Nine Years, at the Rate of* 6 per Cent. per Annum?

Hence *R* = 1,06 its Log. 0.025306 } Multiply
And *t* = 9

Their Product is 0.227754 } Add
Alſo *P*=567,5 its Log. is 2.753966

The Sum is the Logarith. 2.981720 of 958,78 = *A*.

That is, 958 *l.* 15 *s.* 7 *d.* will be the Amount requir'd:

Conſequently 958*l.* 15*s.* 7*d.*— 567*l.* 10*s.*=391*l.* 5*s.* 7*d.* will be the Intereſt only, which is more than the Simple Intereſt of that Principal, for the ſame Time, by 84 *l.* 16 *s.* 7 *d.* *See Page* 23.

Caſe 2. When A, t, and R, are given; To find P.

Rule. *Multiply the Log. of* R *the Amount of* 1 l. *at the given Rate* per Cent. *with the Time* (as before;) *Then Subſtract that Product from the Logar. of the propoſed Amount, and there will Remain the Logarithm of the Principal required.*

Example.

What Principal or Sum of Money, being put out to Intereſt for Nine Years, will Amount to (or Raiſe the Sum of) 958 l. 15 s. 7 d. *at the Rate of* 6 per Cent. per Annum?

Or thus; *There is a Debt of* 958 l. 15 s. 7 d. *which is not due until Nine Years hence, but 'tis agreed to be paid in preſent Money; What Sum muſt the Creditor Receive, allowing the Rebate or Diſcompt, of* 6 per Cent. per Annum *to the Debtor for his ready Money.*

Here $A = 958,78$ its Log. 2.981720
And $R = 1,06$ its Log. is 0.025306 } Multiply.
Alſo $t =$ 9 }
Subſt. this Log. from the firſt 0.227754
There Remains the Logar. 2.753966 of $567,5 = P$.

That is, 567 *l.* 10 *s.* is the Principal (or ready Money) as was required.

Caſe 3. Suppoſe P, t, and A, are given; To find R.

Rule. *From the Log. of the propoſed Amount, Subſtract the Log. of the propoſed Principal; Then Divide the Remainder by the Time, and the Quotient will be the Logarithm of* R, *the Amount of* 1 l. &c. *As in this Example.*

Exam-

Example.

At what Rate of Intereſt per Cent. &c. *will* 567 l. 10 s. *Raiſe a Stock, or Amount to* 958 l. 15 s. 7 d. *in Nine Years Time?*

Here is given $A=958,78$ its Log. 2.981720 } Subſtract
And $P=567,5$ its Logarith. is 2.753966 }

Alſo $t=9$ Then 9) 0.227754 (0,025306 the Log. of $1,06=R$; then $1,06-1=0,06$ the Ratio, And it will be As 1 : 0,06 :: So is 100 : To 6 the Rate of Intereſt *per Cent. &c.* (*See Page* 25.)

Caſe 4. Having P, R, and A, given; To find t.

Rule. { *From the Log. of the propoſed Amount, Subſtract the Log. of the propoſed Principal* (as before) *and Divide their Difference by the Log. of R the Amount of* 1 l. *at the given Rate, and the Quotient will ſhew the Time required.*

Example.

In what Time will 567 l. 10 s. *Amount to, (or Raiſe a Stock of)* 958 l. 15 s. 7 d. *at the Rate of* 6 per Cent. per Annum?

Here is given $A=958,78$ its Log. 2.981720 } Subſt.
And $P=567,5$ its Logarithm is 2.753966 }

Alſo $R=1,06$ its Log. is 0,025306) 0.227754 ($9=t$ *Viz.* Nine Years will be the Time required by the Queſtion.

Theſe Four Examples are ſufficient to ſhew how all Queſtions of the like kind may be truly Reſolved; if the Time given (*or ſought*) be whole Years; but if it be not, then the Odd Time, whether it be under, or above a Compleat Year, muſt be Reduc'd into Decimal Parts of a Year, (*See pag.* 27) by help of the two Tables in Page 28 and 29; *&c.* and then it will be as eaſie to Reſolve any of the foregoing Caſes, when the Time given (or ſought) is in Parts of a Year, as it is for thoſe in whole Years.

Example 1. *When the Time is Leſs then a Year. Let it be required to find the Amount of* 375 l. *for* 210 *Days, at* 6 per Cent. &c.

In this Queſtion there is given, $P=375$ $R=1{,}06$ and $t=210$ Days $=0{,}5753$ Decimal Parts of a Year, *per Table, pag.* 29; Thence to find A; as in *Caſe* 1. of this *Section.*

First $R=1{,}06$ its Log. is 0.025306 } Multiply.
And $t=$ 0.5753 }
Their Product is 0.014558 } Add
Again $P=375$ its Log. is 2.574031 }
The Sum is the Logar. 2.588589 of $387{,}78=A$.
That is 387 *l.* 15 *s.* 7 *d.* will be the Amount requir'd.

Example 2. *Suppoſe it were requir'd to find what* 265 l. *would Amount to in Five Years and* 135 *Days; at the Rate of* 6 per Cent. &c.

In this Queſtion is given $P=265$ $R=1{,}06$ and $t=5{,}3698$ To find A the Amount; As before.

Thus $R=1{,}06$ its Log. 0.025306 } Multiply
And $t=$ 5,3698 }
Their Product is 0.135888 } Add
Again $P=265$ its Log. 2.423246 }
The Sum is the Logar. 2.559134 of $362{,}355=A$.
Viz. 362 *l.* 7 *s.* 1 *d.* will be the Amount or Sum requir'd.

Example 3. *Let it be requir'd to find what Principal, or Sum put to Intereſt at* 6 per Cent. *will Amount to (or Raiſe a Stock of)* 362 l. 7 s. 1 d. *in Five Years and* 135 *Days.*

Here is given $A=362{,}355$ $R=1{,}06$ and $t=5{,}3698$ To find P, as in *Caſe* 2.

Thus $A = 362{,}355$ its Log. 2.559134 of the Divid.
And $R = 1{,}06$ its Log. is 0.025306 } Multiply
$t =$ 5,3698 }
Then Subſtract this Logar. 0.135888 from 1ſt Log.
And there Remains the Log. 2.423246 of $265 = P.$
Viz. 265 *l.* will be the Principal, or Sum requir'd.

If the Work of theſe Three Examples be well underſtood, it muſt needs be eaſie to find proper Anſwers to all Queſtions that can be propoſed in the other Caſes, *viz.* for any propoſed Time, and Rate of Intereſt, according as the Date requires.

Sect. 2. Of **Annuities**, or **Rents**, *&c.* that are in Arrears, Computed at *Compound Intereſt.*

I ſhall here make uſe of the ſame Letter to repreſent the ſame things as in *Page* 32. Save only that R doth here denote the Amount of 1 *l.* As in the laſt Section.

Viz. { $U =$ the Annuity, or Rent.
$t =$ the Time of its being unpaid.
$A =$ the Amount, or Sum of all the Arrears.
And $R =$ the Amount of 1 *l.* *&c.* as in *Page* 53.

$$\text{Then will } A - U = AR - UR^t.$$

From this Æquation is deduced the Four following Caſes, and the Rules by which they are Reſolved.

N. B. *The following Caſes admits of ſome Variety, in reſpect to the true Intervals of the Times, which are betwixt the ſeveral Payments*; viz. *it muſt be well conſidered whether the Payments are to be made Yearly, or at the End of ſome part of a Year*; *As the* $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{12}$. $\frac{1}{52}$. $\frac{1}{365}$. *That is*, Half-yearly, Quarterly, Monthly, &c. *the which muſt be very carefully minded when the Queſtion is propoſed, in any of the following Caſes, or the Anſwers will not be true.*

Caſe 1. Having *U*, *t*, and *R*, given; To find *A*.

That is, If any Annuity, or Rent, *&c.* with the Time of its being unpaid, and the Rate of Intereſt *per Cent.* are given; Thence to find what Sum all thoſe Arrears will Amount to in that Time, allowing Compound Intereſt for every particular Payment, as it becomes due.

Rule. *Multiply the Log. of R (the Amount of* 1 l.) *with the Time; and to that Product Add the Log. of the Annuity; Then find the Number which belongs to that Sum, and Subſtract the Annuity from that Number;* (So far the Work is general to all Times of Payment.) *Then, if the Remainder* (*) *be Divided by R* — 1 (viz. *by the Ratio of the Rate of Intereſt* per Cent.) *the Quotient will ſhew the Sum, or Amount of all the Arrears for Yearly Payments.*

Example 1.

Suppoſe 356 l. *Annuity, or Yearly Rent, were forborn or unpaid Nine Years; What would the Sum of all thoſe Arrears Amount to at* 6 per Cent. &c. ?

Here is given $U = 356$ $t = 9$ and $R = 1{,}06$ To find *A*.

$$\begin{array}{rrl} \text{Thus } R = 1{,}06 \text{ its Log. is} & 0.025306 & \rule{0pt}{0pt}\left.\right\} \\ t = & 9 & \left.\right\} \text{Multiply} \\ \hline \text{The Product is} & 0.227754 & \left.\right\} \text{Add} \\ \text{Again } U = 356 \text{ its Log. is} & 2.551450 & \left.\right\} \\ \hline \text{The Sum is the Logarithm} & 2.779204 & \text{of } 601{,}4545 \end{array}$$

And $601{,}4545 - 356 = 245{,}4545$; $R - 1 = 0{,}06$
Then $0{,}06)245{,}4545(4090{,}91 = A = 4090$*l.* 18*s.* $2\frac{1}{2}$*d.* being the Sum of all the Arrears for Yearly Payments.

Note, *Here we may obſerve the Difference that is betwixt the Arrears Computed at Simple Intereſt, and at Compound Intereſt; See the ſame Example in Page* 33.

Or, the Amount may be alſo found by Logarithms without the laſt Diviſion; for having once form'd the Dividend, and Diviſor (as before) then the Difference of their Logarithm will ſhew the Amount.

Thus 245,4545 its Log. 2.389972 } Subſtract
And $R-1=0,06$ its Log. $\bar{8}$.778151 }
There Remains the Log. 3.611821 of 4090,91$=A$.
And ſo on, As before.

But if the Payments are to be made at any Time Leſs than a compleat Year; As $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{12}$. &c.

Then { *You muſt take the like part of the aforeſaid Remainder* (*) *and make it a new Dividend: And you muſt alſo take the ſame part of the Logarithm of R; and the Number anſwering to that part being made leſs by an Unit or* 1, *will be a new Diviſor* (*inſtead of* $R-1$) *by which the new Dividend being Divided, the Quotient will ſhew the Sum of all the Arrears required.*

Example 2. In Half-yearly Payments.

Suppoſe the aforeſaid 356 l. per Annum, *were to be paid by Half-yearly Payments,* viz. 178 l. *every Half-Year, and it were unpaid Nine Years* (as before) *What Sum would all thoſe Arrears Amount to then, at* 6 per Cent. &c.?

Here (*as bef.*) $R=1,06$ itsLog. 0.025306 } Multiply
And $t=$ 9 }
Their Product is 0.227754 } Add
Alſo $U=356$ its Log. is 2.551450 }
The Sum is the Logarithm 2.779204 of 601,4545

and $601,4545-356=245,4545$. So far the Work is juſt the ſame as before for Whole Years:

But here the $\frac{1}{2}$ of $245,4545=122,72725$ is the New Dividend:

Next, $R =$ 1,06 the ½ of its Log. is 0.012653 whoſe Number is 1,029563 and 1,029563 — 1 = 0,029563 for the New Diviſor;

Then 0,029563) 122,72725 (4151,38 = A.

That is, 4151 *l.* 7 *s.* 7 *d.* is now the Amount or Sum of all the Arrears; which is 60 *l.* 9 *s.* 4¼ *d.* more than the Amount in the laſt Example for whole Yearly Payments.

Here alſo the Amount may be obtain'd by Logarithms without the laſt Diviſion; having once form'd the New Dividend, and its proper Diviſor as before:

Thus, 122,727 its Log. is 2.088938 } Subſtract
And 0,029563 its Log. is 8.470748 }
There Remains the Log. 3.618190 of 4151,38 = A
Viz. 4151 *l.* 7 *s.* 7 *d.* = A the Amount, *&c.* As before.

Example 3. In Quarterly Payments.

Let it be required to find the Amount, or Sum of all the Arrears that would be due from 245 l. per Annum, *to be paid by Quarterly Payments* (viz. 61 l. 5 s. per Quarter) *ſuppoſing it to be forborn, or unpaid Ten Years and a Half, at* 5½ per Cent. &c.

Here is given $R =$ 1,055 its Log. 0.023252 } Multiply
And $t =$ 10,5 }
The Product is 0.244146 } Add
Again $U =$ 245 its Log. is 2.389166 }
Their Sum is the Logarith. 2.633312 of 429,846

And 429,846 — 245 = 184,846 So far the Work is ſtill the ſame as in whole Years.

But now it muſt be the ¼ of 184,846 = 46,2115 for the New Dividend.

Next, $R =$ 1,055 the ¼ of its Log. is 0.005813 whoſe Numb. is 1,01348 and 1,01348 — 1 = 0,01348 for the New Diviſor.

Then 0,01348) 46,2115 (3428,15 = *A.*

That is, 3428 *l.* 3 *s.* will be the Amount or Sum of all thoſe Arrears, as was required.

Or the laſt Diviſion may be avoided, and the Amount may be found by the Logarithms; Thus,

Dividend 46,2115 its Log. 1.664750 } Subſtract
Diviſor is 0,01348 its Log. 8.129690 }
There Remains the Logar. 3.535060 of 3428,15 = *A*
Viz. 3428 *l.* 3 *s.* is the Amount requir'd, as above.

The ſame may be done for *Monthly*, *Weekly*, or Daily Payments, Care being taken in forming the New Dividend, and Diviſor, according to their reſpective Time.

Caſe 2. When *A*, *t*, and *R*, are given; To find *U*. That is, To find what Annuity, or Yearly Rent being forborn or unpaid, any aſſigned Time, will Amount to a propoſed Sum, allowing any given Rate of Intereſt *per Cent.* for every Payment as it becomes due.

Note, *This Caſe, as well as the Laſt, admits of ſome Variety in reſpect to the Times of Payment*; viz. *Whether they are* Yearly, Half-yearly, *or* Quarterly, &c. *the which muſt be carefully obſerved, as the following* Rule *directs.*

Rule. { *Multiply the Log. of R* (the Amount of 1 *l.*) *with the given Time, and the Number anſwering to their Product being made leſs by* 1, *will be the Diviſor*, (for all Times of Payment at that Rate of Intereſt.)
Next, Multiply the propoſed Amount, or Sum, with R — 1, viz. *with the Ratio of the given Rate; Then Divide their Product by the aforeſaid Diviſor, and the Quotient will ſhew the Annuity, or Rent for Yearly Payments.*

Example 1.

What Annuity, or Yearly Rent, being forborn or unpaid Nine Years, will Amount to (or Raiſe a Stock of) 4090 l. 18 s. 2½ d. *at the Rate of* 6 per Cent. *for every Payment as it becomes Due?*

Here is given $A = 4090,91$ $t = 9$, and $R = 1,06$
To find U.

Firſt $R = 1,06$ its Log. is 0.025306 } Multiply
And $t = 9$

The Product is the Logar. 0.227754 of 1,68948
And 1,68948 — 1 = 0,68948 for the Diviſor.

Again $A = 4090,91$ } Multiply
And $R - 1 = 0,06$

The Product is 245,4546 for the Dividend.

Then 0,68948) 245,4546 ($356 = U$.

That is 356 *l.* will be the Annuity, or Yearly Rent, as was requir'd to be paid by Yearly Payments.

Or having found the Diviſor (*as above*) the reſt of the Work may be perform'd by Logarithms only;

Thus, $A = 4090,91$ its Log. 3.611821 } Add
And $R - 1 = 0,06$ its Log. 8.778151

Sum is 2.389972 } Subſtract
The Diviſ. is 0,68948 its Log 9.838522

There Remains the Logarith. 2.551450 of $356 = U$.

Viz. 356 *l.* will be the Yearly Annuity, or Rent, &c.

But if the Payments are required to be paid either Half-yearly, or Quarterly, &c. As in the laſt Caſe;

Then { *Inſtead of* $R - 1$, *you muſt take the reſpective Part of the Logarithm of* R (viz. of the Amount of 1 *l.*) *and with the Number anſwering to that Part, being made leſs by* 1, *Multiply the propoſed Amount; Then Divide that Product by the aforeſaid Diviſor, and the Quotient will ſhew what Annuity, or Rent muſt be paid at the propoſed Time of each Payment.* }

Exam-

Example 2. In Half-yearly Payments.

What Annuity, or Rent, to be paid by Half-yearly Payments, being forborn or unpaid Nine Years, will Amount to the Sum of 4151 l. 7 s. 7 d. *allowing* 6 per Cent. *for every Payment as it becomes Due?*

Here is given $A = 4151,38$ $t = 9$, and $R = 1,06$
To find U.

Firſt $R = 1,06$ its Log. is 0.025306 } Multiply
And $t =$ 9 }
The Product is the Log. 0.227754 of 1,68948
And 1,68948 — 1 = 0,68948 for the Diviſor, as before in the laſt Example.

But inſtead of the Multiplicator $R - 1$, it muſt be $R = 1,06$ the $\frac{1}{2}$ of its Log. 0,012653 its N°. is 1,029563 and 1,029563 — 1 = 0,029563 the Multiplicator.

And it muſt be $A =$ 4151,38
Multiplied with 0,029563
The Product is 122,72724 the New Dividend.

Then 0,68948) 122,72724 (178 = U, viz. 178 *l.* will be the Half-yearly Payments of the Annuity, or Rent required.

Or having found the Diviſor, and the New Multiplicator uſed inſtead of $R - 1$; Then the Work may be done by the Logarithms,

Thus, $A = 4151,38$ its Log. 3.618190 } Add
Multipl. is 0,029563 its Log. 8.470749 }
Sum 2.088939 } Subſtract
Diviſor is 0,68948 its Log. is 9.838520 }
There Remains the Logarith. 2.250419 of 178 = U
Viz. 178 *l.* is the Half-yearly Payments, *&c.* As before.

Example 3. In Quarterly Payments.

Let it be required to find what Annuity, or Rent to be paid by Quarterly Payments, being forborn or unpaid Ten

Ten Years and a Half, would Amount to the Sum of 3428 l. 3 s. *at Five and a Half* per Cent. per Annum, *for every Payment as it becomes Due?*

Here is given $A = 3428{,}15$ $t = 10{,}5$ and $R = 1{,}055$
To find U, the Quarterly Payments.

Firſt $R = 1{,}055$ its Log. is 0.023252 } Multiply
And $t =$ 10,5 }

The Product is the Logar. 0.244146 of 1,75447
Then $1{,}75447 - 1 = 0{,}75447$ the Diviſor.

Again $R = 1{,}055$ the $\frac{1}{4}$ of its Log. is 0.005813 its Number is 1,01348 and $1{,}01348 - 1 = 0{,}01348$ the Multiplicator.

Next $A = 3428{,}15$
Multiplied with 0,01348
Produces 46,21146 the Dividend.

Then 0,75447) 46,21146 ($61{,}25 = U$.

That is, 61 *l.* 5 *s.* will be the Quarterly Rents or Payments; conſequently $61{,}25 \times 4 = 245$ *l. per Annum* will be the Annuity requir'd.

Or by *Logarithms*, Thus

$A = 3428{,}15$ its Log. is 3.535060 } Add
And 0,01348 its Log. is $\bar{8}.129690$ }
Sum is 1.664750 } Subſtract
Again 0,75447 its Log. $\bar{9}.877642$ }
There Remains the Log. 1.787108 of $61{,}25 = U$

Viz. 61 *l.* 5 *s.* will be the Annuity, or Rent to be paid every Quarter, *&c.*

The like may be done in finding the Monthly, Weekly, or Daily Annuities, care being taken in Forming the New Multiplicators, and Diviſors proper for the Time given.

Caſe 3. Having U, A, and R given; To find t. That is, To find the Time in which any propoſed Annuity, or Rent will Amount to any aſſign'd Sum, at any given Rate of Intereſt *per Cent.* &c.

Rule. *Multiply the propoſed Amount with* R — 1, viz. *with the Ratio of the given Rate ; and Divide that Product by the Annuity, and to the Quotient Add* 1. *Then if the Logarithm of that Sum be Divided by the Logarithm of R* (*viz* the Amount of 1 *l.*) *the Quotient will ſhew the Time for Yearly Payments.*

Example 1.

In what Time will 356 l. *Yearly Rent, or Annuity, Amount to the Sum of* 4090 l. 18 s. $2\frac{1}{2}$ d. *at the Rate of* 6 per Cent. &c.

Here is given $U=356$, $A=4090,91$ and $R=1,06$
To find t.

$$\left.\begin{array}{ll}\text{Thus,} & A = 4090,91 \\ \text{And} & R - 1 = 0,06\end{array}\right\}\text{Multiply}$$

$U = 356$) 245,4546 (0,68948

And $0,68948 + 1 = 1,68948$ its Log. is 0,227754

Then $R = 1,06$ its Log. 0,025306) 0,227754 ($9 = t$. *Viz.* 9 Years will be the Time requir'd in the Queſtion ;

Or otherwiſe by *Logarithms*,

$$\left.\begin{array}{lr}\text{Thus } A = 4090,91 \text{ its Log. is} & 3.611821 \\ \text{And } R - 1 = 0,06 \text{ its Log.} & \bar{8}.778151\end{array}\right\}\text{Add}$$

$$\left.\begin{array}{lr}\text{Sum} & 2.389972 \\ \text{Again } U = 356 \text{ its Log. is} & 2.551450\end{array}\right\}\text{Subſtract}$$

There Remains the Logar. $\bar{9}.838522$ of 0,68948

And $0,68948 + 1 = 1,68948$ its Log. 0,227754. Then *as before*, 1,06 its Log. 0,025306) 0,227754 ($9 = t$. *Viz.* 9 Years will be the Time required, if the Payments are to be paid but once a Year.

But if they are to be paid either *Halfyearly*, or *Quarterly*, &c. As before in the two laſt Caſes ;

Then { *Inſtead of making* R — 1, *the Multiplicator as above, you muſt take ſuch a Part of the Logarithm of* R *the Amount of* 1 l. *as the given Time requires; and with its Number made Leſs by* 1, *Multiply the propoſed Amount; Then Divide that Product by the Sum which ought to be paid at its reſpective Time; and to that Quotient Add* 1; *and ſo proceed as the* Rule *directs.*

Example 2. In Half-yearly Payments.

In what Time will 356 l. per Annum *Amount to the Sum of* 4151 l. 7 s. 7 d. *If it is to be paid by Half-yearly Payments*, viz. 178 l. *every Half-Year; and to be allow'd* 6 per Cent. &c. *for every Payment as it becomes Due?*

Here is given $U=178$ *viz.* the $\frac{1}{2}$ of 356, $A=4151,38$ and $R=1,06$ To find t, the Number of Years.

Firſt $R=1,06$ the $\frac{1}{2}$ of its Log. is 0.012653 and its Numb. is 1,029563 then $1,029563 - 1 = 0,029563$ the Multiplicator inſtead of $R - 1$.

And it will be $A = 4151,38$
Multiplied with 0,029563

The Product is 122,72724 for the Dividend.

Then $U = 178$) 122,72724 (0,68948 to which Add 1, and it will be 1,68948 its Log. is 0,227754

And $R = 1,06$ its Log. is 0.025306) 0,227754 ($9 = t$

Viz. 9 Years will be the Time ſought.

Or having once form'd the Multiplicator, then the Work may be done by Logarithms,

Thus $A = 4151,38$ its Log. is $\underline{3}.618190$ } Add
Multiplic. is 0,029563 itsLog. 8.470749 }

Sum 2.088939 } Subſtract
Again $U = 178$ its Log. is 2.250418 }

There Remains the Logarith. $\overline{9}.838521$ of 0,68948.

And 0,68948 + 1 = 1,68948 its Log. is 0.227754
R = 1,06 its Log. is 0.025306) 0.227754 (9 = t, the Number of Years ſought. As before.

Example 3. In Quarterly Payments.

In what Time will 245 l. per Annum, *to be paid Quarterly* (viz. 61 l. 5 s. per *Quarter*) *Amount to the Sum of* 3428 l. 3 s. *allowing* $5\frac{1}{2}$ per Cent. *for the Forbearance of the Payments as they become Due?*

Here is given U = 61,25 (*viz.* the $\frac{1}{4}$ of 245 *l.*) R = 1,055
and A = 3428,15 To find t, the Time.

Firſt R = 1,055 the $\frac{1}{4}$ of its Log. is 0.009813 its Number is 1,01348, and 1,01348 − 1 = 0,01348 the *Multiplicator*; And A = 3428,15 the *Multiplicand*,
Multiplied with 0.01348
Their *Product* is 46,21146 the *Dividend*.

Then U = 178) 46,21146 (0,75447 to which
Add 1, and it is 1,75447 its Log. is 0.244146:

Then R = 1,055 its Log. 0.023252) 0.244146 (10,5 = t
Viz. Ten Years and a Half will be the Time required by the Queſtion.

The ſame may be found by *Logarithms*, as in the laſt Example.

Caſe 4. Having U, t, and A, given; To find R.

That is, If any Annuity, with the Time of its being unpaid, and the Sum it's propoſed to Amount to in that Time, are given; Thence to find the Rate of Intereſt *per Cent.* that muſt be allow'd for every Payment as it becomes Due.

The Solution of this Queſtion is more difficult than any hitherto has been; And the Æquation by which the Value of R muſt be found is this, $\frac{A}{U} - 1 = \frac{A}{U} R - R^t$, which is called an Adfected Æquation, and requires an Analytical Solution to find the Value of *R*, (*as in the Young Mathematician's Guide*, Pag. 268.) which cannot otherwiſe be found by any General *Rule*, that can be preſcrib'd here, as in the precedent Caſes, but only by Approaching to it with Trials. And in order to perform that Approximation with the leaſt Trouble, I ſhall here propoſe the following Method, as the eaſieſt I can at preſent think of.

Which is thus:

Firſt Divide the propoſed Amount, by the Annuity, and Reſerve the Quotient. Next Subſtract 1 *from that Quotient, and call the Remainder* x.

Then you muſt make choice (by gueſs) *of ſuch a Number for the Value of* R, *as you think will be the Amount of* 1 l. *at the Rate of the Intereſt ſought; and Multiply that Number into the reſerved Quotient; Alſo you muſt Multiply the Logarithm of that ſuppoſed Number, with the Time; and find the Number which belongs to their Product: Then, if the Difference between that Number and the aforeſaid Product of* R *into the Reſerved Quotient, be Equal to* (*viz.* be the ſame with) *the Number call'd* x, *you have hit upon the true Value of* R. *But if they are not Equal, then another Trial muſt be made*, &c. *As in the following Examples.*

Example 1. In Yearly Payments.

There is an Annuity, or Rent of 356 l. per Annum, *and it has been forborn or unpaid Nine Years; And there is* 4090 l. 18 s. $2\frac{1}{2}$ d. *given for the Amount, or Sum of all the Arrears; it's required to find what Rate of Intereſt* per Cent. *is allow'd for each Payment as it became Due?*

Here is given $U=356$ $t=9$, and $A=4090{,}91$
To find *R*, the Amount of 1 *l.*

Thus 356) 4090,91 (11,49132 the *Quotient* to be Reſerv'd; from which Subſt. 1, and it will be 10,49132, which is the Number call'd *x*: And ſo far the Work will always be certain, it being only that of the firſt Side of the Æquation.

Next, I will ſuppoſe the Value of R to be 1,05 and then the Reſerved Quotient 11,49132 × 1,05 the Product will be 12,065886.

Again, If $R = 1,05$ its Log. 0.021189 } Multiply
And $t = 9$ }

The Product is the Logar. 0.190701 of 1,55132

Then 12,065886 — 1,55132 = 10,51456 which is More than 10,49132 the Number called x; Therefore this ſuppoſed Value of $R = 1,05$ is taken too Little.

Again, For a ſecond Trial I will ſuppoſe $R = 1,07$ and then the Reſerved Quotient 11,49132 × 1,07 the Product will be 12,29571.

And if $R = 1,07$ its Log. is 0.029384 } Multiply
And $t = 9$ }

The Product is the Logar. 0.264456 of 1,83847

Then 12,29571 — 1,83847 = 10,45724 which is Leſs than 10,49132 the Num. called x: And therefore I conclude the Laſt $R = 1,07$ was taken too Big, and that the Value of R is ſome Num. between 1,05 and 1,07

Let us therefore take $R = 1,06$ (*viz. in the Middle between* 1,05 *and* 1,07.) And then the Reſerved Quotient 11,49132 × 1,06, the Prod. will be 12,180799

And if $R = 1,06$ its Log. is 0.025306 } Multiply
And $t = 9$ }

The Product is the Logar. 0.227754 of 1,689479

Then 12,180799 — 1,689479 = 10,49132 which is juſt the ſame with the Number called x.

Whence I conclude that the true Value of $R = 1,06$ and 1,06 — 1 = 0,06 is the Ratio of the Rate of Intereſt ſought.

Then it will be, As 1 : Is to 0,06 :: So is 100 : To 6: the Rate of Intereſt *per Cent.* As was requir'd for Yearly Payments.

But if the Payments are to be made either Half-yearly, or Quarterly, &c.

Then $\left\{\right.$ *Inſtead of Multiplying the Reſerved Quotient, with the ſuppoſed Value of R* (as above) *you muſt take ſuch a Part of the Log. of that R* (viz. of the aſſumed Amount of 1*l.*) *as the given Time requires, and with its Number Multiply the aforeſaid Reſerv'd Quotient, and then proceed on as before.*

Example 2. In Half-yearly Payments.

Suppoſe the ſame Annuity of 356 l. per Annum *were to be paid Half-yearly* (*viz.* 178 *l* every Half-Year) *If it be forborn Nine Years, and then Amounts to the Sum of* 4151 l. 7 s. 7 d. *What Rate of Intereſt* per Cent. *is allow'd,* &c. ?

Here is given $U = 178$ (viz. *the* ½ *of* 356 l.) $t = 9$ and $A = 4151{,}38$ To find R, the Amount of 1 *l.* &c.

Firſt 178) 4151,38 (23,32236 the Reſerved Quot. and 23,32236 — 1 = 22,32236 the Number called *x*. *So far is the ſame as before in whole Years.*

Now here I will aſſume the firſt Value of $R = 1{,}06$ Then if $R = 1{,}06$ the ½ of its Log. 0.012653 and the Number to that Half Log. is 1,029563 the Multiplicator inſtead of $R = 1{,}06$ as before: And then the Reſerved Quotient $23{,}32236 \times 1{,}0295663$, the Product will be 24,011839.

$$\begin{array}{lr} \text{Again, If } R = 1{,}06 \text{ its Log is} & 0.025306 \\ \text{And } t = & 9 \end{array}\Bigg\}\text{Multiply}$$

$$\text{Their Product is the Logar. } \overline{0.227754} \text{ of } 1{,}689479$$

Then $24{,}011839 - 1{,}689479 = 22{,}32236$ which is juſt the ſame with 22,32236 the Number called *x*. And therefore the true Value of $R = 1{,}06$ And ſo on for the Rate *per Cent.* as in the laſt Example.

Example

Example 3. In Quarterly Payments.

An Annuity of 245 l. per Annum, *to be paid Quarterly* (*viz.* 61 *l.* 5 *s.* every Quarter) *is forborn Ten Years and a Half*; *and then 'tis ſaid to Amount to the Sum of* 3428 l. 3 s. *It is required to find what Rate of Intereſt* per Cent. *is allow'd*, &c.

Now here is given $U = 61{,}25$ (viz. *the* ¼ *of* 245 l.) $t = 10{,}5$ and $A = 3428{,}15$ To find R, the Amount of 1 *l.* &c.

Firſt $61{,}25\ )\ 3428{,}15\ (55{,}96979$ the Reſerved Quot.
and $55{,}96979 - 1 = 54{,}96979$ the Numb. called x.

Then let us here ſuppoſe $R = 1{,}05$ and then the ¼ of its Log. will be 0.005297 whoſe Num. is 1,01227 the Multiplicator.

And the Reſerved Quotient $55{,}96979 \times 1{,}01227$ the Product will be 56,65754.

$$\left.\begin{array}{r}\text{Again, If } R = 1{,}05 \text{ its Log. is } 0.021189\\ \text{And } t = 10{,}5\end{array}\right\}\text{Multiply}$$

The Product is the Logarith. 0.222484 of 1,66911

Then $56{,}65754 - 1{,}66911 = 54{,}98843$ which is a little More than 54,96979 the Number called x. Therefore the Value of $R = 1{,}05$ is taken ſomething too Little.

And making a ſecond Trial, by ſuppoſing $R = 1{,}06$ and then proceeding on as before, I find the Reſult to be about as much Leſs than 54,96979 the Number called x, as the laſt found Number was too much.

I ſhall therefore aſſume $R = 1{,}055$ *viz.* in the Middle between 1,05 and 1,06

And then if $R = 1{,}055$ the ¼ of its Log. 0.005813 and its Number is 1,01348 for the Multiplicator.

Then the Reſerved Quotient $55{,}96979 \times 1{,}01348$ the Product is 56,72426.

Again, If $R = 1{,}055$ its Log. is 0.023252 } Multiply
And $t =$ 10,5 }

The Product is the Logarith. 0.244146 of 1,75447

Then 56,72426 — 1,75447 = 54,96979 which is exactly the ſame with the Number called x.

And therefore I conclude that the true Value of $R = 1{,}055$ and 1,055 — 1 = 0,055 the Ratio of the Rate. And

Then it will be As 1 : Is to 0,055 :: So is 100 : To 55, *Viz.* Five and a Half is the Rate of Intereſt *per Cent. per Annum*, as was required.

What hath been here done by Multiplication, and Diviſion, may, if you pleaſe, be perform'd by Logarithms, as in the precedent Caſes.

As for Inſtance in the laſt Example;

Wherein $A = 3428{,}15$ its Log. 3.535060 } Subſtract
And $U = 61{,}25$ its Log. is 1.787108 }

The Remainder is the Log. 1.747952 of 55,9698 which is here the Reſerved Quotient.

And 55,9698 — 1 = 54,9698 the Number called x.

Then if $R = 1{,}055$ the ¼ of its Log. 0.005813 } Add
the Log. of the Reſerv'd Quotient 1.747952 }

The Sum is the Logarithm 1.753765 of 56,7243

Again, If $R = 1{,}055$ its Log. is 0.023252 } Multiply
And $t =$ 10,5 }

The Product is the Logarithm 0.244146 of 1,75447

Then 56,7243 — 1,75447 = 54,96983 is juſt the ſame with the Number called x; And therefore, &c. As above.

Sect. 3. To find the Preſent Worth of Annuities, Penſions, or Leaſes, &c. at Compound Intereſt.

I ſhall here make uſe of the ſame Letters to repreſent the ſeveral Parts of the Queſtion, as in *Sect.* 3. *Pag.* 39. And it may not be amiſs to make uſe alſo of the ſame *Examples* as are in that *Section*; ſave only that I ſhall here Compute them for whole Years, Half-Years, and Quarterly Payments, as in the laſt *Section.*

Let { *U.* Denote the *Annuity*, or *Rent*.
t. The *Time* of its *Continuance*.
R. The *Amount* of 1 *l*. } As before;
And *P.* The *Preſent Worth* of the *Annuity*, *&c.*

Then will $PR^t = \frac{UR^t - U}{R - 1}$.

(Vide *the Young Mathematician's Guide*, Page 269.)

From this Æquation is deduc'd the following Rules, which alſo admit of Four Caſes.

Caſe 1. Having *U*, *t*, and *R*, given; To find *P*

That is, If any propoſed Annuity, or Yearly Rent, and the Time of its Continuance are given; To find the Preſent Worth of that Annuity, allowing any Rate of Compound Intereſt *per Cent.* to the Purchaſer, *&c.*

Rule. { *Multiply the Log. of* R (the Amount of 1 *l.*) *with the given Time*; *and from the Number which belongs to that Product, Subſtract* 1 : *Then Multiply the Remainder with the Annuity, and the Product will be a Dividend.*

Then Multiply the Number which belong'd to the firſt Product, with R — 1 (*viz.* with the Ratio of the Rate) *and make their Product a Diviſor*; *The Quotient ariſing from thence will ſhew the Preſent Worth for Yearly Payments.* }

Example 1. In Yearly Rents.

Suppoſe a Leaſe of 250 l. per Annum, *were to be Lett for* 21 *Years; What may the preſent Worth of that Leaſe be, at the Rate of* 5 per Cent. &c.?

In this Queſtion there is given $U = 250$, $t = 21$, and $R = 1{,}05$ To find P, the preſent Worth for Yearly Rents.

First $R = 1{,}05$ its Log. is 0.021189 } Multiply
And $t =$ 21 }

The Product is the Logar. 0.444969 of 2,7859
Then $2{,}7859 - 1 = 1{,}7859$ And $1{,}7859 \times 250 = 446{,}475$ for the Dividend.

Again $R - 1 = 0{,}05$ And $2{,}7859 \times 0{,}05 = 0{,}139295$ for the Diviſor.

Then 0,139295) 446,475 ($3205{,}2478 = P$.

That is, 3205 *l.* 5 *s.* will be the preſent Worth of ſuch a Leaſe as was required.

Or the ſame may be otherwiſe performed by *Logarithms*: Thus,

Firſt $R = 1{,}05$ its Log. is 0.021189 } Mult. as before.
And $t =$ 21 }

The Product is 0.444969 } Add
Again $U = 250$ its Log. 2.397940 }

The Sum is the Logar. 2.842909 of 696,48 from which Subſtract the Annuity 250 and there will Remain 446,48 its Logar. is 2.649802

The Log of R, Mult. with *t*, is 0.444969 } Add
And $R - 1 = 0{,}05$ its Log. is 8.698970 }

From the up. Log. take this Log. 9.143939

There Remains the Logarith. 3.505863 of 3205,25

That is, 3205 *l.* 5 *s.* will be the preſent Worth requir'd by the Queſtion, as before; ſuppoſing the Rents to be paid but once a Year.

But if the Payments of the Annuity, or Rents, *&c.* are to be made at any Time Leſs than a compleat Year, As $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{12}$ *&c.* That is, *Half-yearly*, or *Quarterly*, &c. Then the Diviſor muſt be otherwiſe form'd than by $R - 1$.

Thus { *You muſt take ſuch a part of the Log.* R *the Amount of* 1 l. *as the given Time requires, and make uſe of the Number anſwering to that Part, made leſs by* 1, *inſtead of* R — 1 *the Ratio*, &c. *As in the following Examples.* }

Example 2. For Half-yearly Rents.

Suppoſe the aforeſaid 250 l. per Annum, *were to be paid Half-yearly (viz.* 125 l. every Half-year*) and a Leaſe of it were to be Lett for* 21 *Years; What would the preſent Worth of that Leaſe be, at the Rate of Five* per Cent. &c.

Here is given $U = 125$, $t = 21$, and $R = 1{,}05$ To find P.

Firſt $R = 1{,}05$ its Log. is 0.021189 } Multiply
And $t =$ 21 }

The Product is the Logarith. 0.444969 of 2,7859

Then $2{,}7859 - 1 = 1{,}7859$ and $1{,}7859 \times 125 = 223{,}2375$ for a Dividend. Which is only the Half of the Dividend in the laſt Example.

Next $R = 1{,}05$ the $\frac{1}{2}$ of its Log. is 0.010594 whoſe Number is 1,0247 and $1{,}0247 - 1 = 0{,}0247$ to be the Multiplicator inſtead of $0{,}05 = R - 1$.

That is, $2{,}7859 \times 0{,}0247 = 0{,}06881$ will be the Diviſ.

Then $0{,}06881\,)\;223{,}2375\;(\;3244.25 = P$.

Viz. 3244 *l.* 5 *s.* will be now the preſent Worth of that Leaſe, which is more by 39 *l.* than that in the laſt Example for Yearly Payments.

And by the way, we may here obſerve the great Difference that is betwixt the preſent Worth of any Annuity, or Leaſe, &c. *Computed at* Simple Intereſt, *and the ſame at* Compound Intereſt. *See the Example in Page* 40.

The ſame may alſo be performed by *Logarithms* as in the laſt Example, having found the Multiplicator inſtead of *R* — 1, As above.

Then *R* = 1,05 its Log. is 0.021189 } Multiply
And *t* = 21 }

0.444969 } Add
Again *U* = 125 its Log. is 2.096910 }

The Sum is the Logarithm 2.541879 of 348,24

From this 348,24 Subſtract the Rent 125, and there Remains 223,24 its Logarith. is 2.348772

The Log. of *R*, Multip. with *t*, is 0.444969 } Add
The new Multip. 0,0247 its Log. $\bar{8}$.392697 }

From the up. Log. Subſt. this Log. $\bar{8}$.837666

There Remains the Logarithm 3.511106 of 3244,2

Viz. 3244 *l.* 4 *s.* is here found to be the preſent Worth, which is leſs than that found by the Pen, by 1*s.* being but very inconſiderable.

Example 3. In Quarterly Rents.

What is 80 l. per Annum, *to continue* 31 *Years and a Half, Worth in preſent Money, at the Rate of* 7 per Cent. per Annum; *If the Rents be paid Quarterly,* (*viz.* 20 *l.* every Quarter of a Year?)

In this Queſtion *U* = 20, *t* = 31,5 and *R* = 1,07
To find *P*, As before.

Firſt *R* = 1,07 its Log. is 0.029384 } Multiply
And *t* = 31,5 }

The Product is the Logarith. 0.925596 of 8,4255

And 8,4255 — 1 = 7,4255 and 7,4255 × 20 = 148,51 for the Dividend.

Again R=1,07 the $\frac{1}{4}$ of its Log. is 0.007346 whoſe Number is 1,01706 and 1,01706 — 1 = 0,01706

Next 8,4255 × 0,01706 = 0,143739 the Diviſor.

Then 0,143739) 148,510000 (1033,185 = P.

That is 1033 *l.* 3 *s.* 8$\frac{1}{2}$ *d.* will be the preſent Worth as was required.

Or having found the Multiplicator 0,01706 As above; Then the reſt of the Work may be perform'd by *Logarithms.*

Thus, R=1,07 its Log. is 0.029384 } Multiply
And t = 31,5 }
0.925596 } Add
Again U=20 its Log. is 1.301030 }
The Sum is the Logarith. 2.226626 of 168,51
From this 168,51 Subſtract the Quarterly Rent 20,
Remains 148,51 its Log. is 2.171756
Log. of R Multiplied with t is 0.925596 } Add
The Multiplic. 0,01706 its Log. $\bar{8}$.231979 }
From 1ſt Log. Subſt. this Log. $\bar{9}$.157575
There Remains the Logarith. 3.014181 of 1033,18

Viz. 1033 *l.* 3 *s.* 7 *d.* is the preſent Worth found by the Logarithms.

Caſe 2. When P, t, and R, are given; To find U That is, To find what Annuity, or Leaſe, &c. may be Purchaſed for any propoſed Sum; to continue any aſſign'd Time, at any given Rate of Intereſt.

Rule.

Firſt Multiply the Logarithm of R *with the Time, and find the Number which belongs to the Product, and call that Number x,* (as in the laſt Section.)

Again, Multiply the ſame Log. of R *with the Time more* 1, *and find the Number which belongs to that Product. Then Multiply the Difference of thoſe Two Num-*

Numbers with the propoſed preſent Worth, and their Product will be a Dividend.

And the firſt Number called x, *being made Leſs by* 1, *will be the Diviſor, by which if the aforeſaid Dividend be Divided, the Quotient will ſhew the Annuity requir'd for Yearly Payments.*

Example 1. For Yearly Rents.

What Annuity, or Yearly Rent, to continue Twenty one Years, may be Purchaſed for 3205 l. 5 s. *at the Rate of Five* per Cent. per Annum?

Here is given $P=3205,25$ $t=21$ and $R=1,05$ To find U.

$$\left.\begin{array}{l}\text{Firſt } R=1,05 \text{ its Log. is } 0.021189\\ \qquad\qquad\qquad \text{And } t= \quad 21\end{array}\right\}\text{Multiply}$$

The Product is the Log. 0.444969 of 2,7859 call'd x.

$$\left.\begin{array}{l}\text{Again } R=1,05 \text{ its Log. is } 0.021189\\ \qquad\qquad\qquad \text{And } t+1= \quad 22\end{array}\right\}\text{Multiply}$$

Their Product is the Logar. 0.466158 of 2,9252

Then $2,9252-2,7859=0,1393$ the Multiplicator.

Next $P=3205,25\times0,1393$ gives 446,4913 for the Dividend.

And the Number called x $2,7859-1=1,7859$ is the Diviſor.

Then 1,7859) 446,4913 (250 l. $=U$ the Annuity requir'd, for Yearly Payments.

Or having once found the Two firſt Numbers, and by them form'd the Multiplicator, and Diviſor, *as above*, the Multiplication, and Diviſion may be perform'd by *Logarithms*.

$$\left.\begin{array}{l}\text{Thus } P=3205,25 \text{ its Log. } 3.505861\\ \text{Multiplic. } 0,1393 \text{ its Log. } 9.143951\end{array}\right\}\text{Add}$$

$$\left.\begin{array}{l}\qquad\qquad\qquad\quad \text{Sum } 2.649812\\ \text{The Diviſ. } 1,7859 \text{ its Log. } 0.251857\end{array}\right\}\text{Subſtract}$$

Remains the Logarithm 2.397955 of $250=U$

Viz. 250 l. will be the Yearly Annuity, as before: But this Caſe is more Univerſally Reſolved by the following Rule.

Rule 2.

Firſt Multiply the Log. of R the Amount of 1 l. with the Time, and find the Number which belongs to that Product, which call x, (As before.)

Next, Multiply the propoſed preſent Worth with R — 1, viz. *with the Ratio of the given Rate, and Multiply their Product with the firſt Number, called x.*

Then Divide the laſt Product by the Number called x made Leſs by 1, and the Quotient will ſhew the Annuity, or Rent for Yearly Payments, (As before.)

As for Inſtance in the laſt Example, wherein $P = 3205,25$ $t = 21$, and $R = 1,05$ To find U.

Then $R = 1,05$ its Log. 0,021189 } Multiply
And $t =$ 21 }

The Product is the Log. 0.444969 of 2,7859 call'd x.

Next $R - 1 = 0,05$ And $P = 3205,25 \times 0,05 = 160,2625$

Again, $160,2635 \times 2,7859 = 446,753$ the Dividend.
And $2,7859 - 1 = 1,7859$ for the Diviſor.

Then 1,7859) 446,753 ($250 = U = 250$ l. the Annuity, or Rent for Yearly Payments.

The ſame may be perform'd by *Logarithms*.

Thus, If $R = 1,05$ and $t = 21$; Then 2,7859 the Number called x will be fonnd as before.

And if $P = 3205,25$ its Log. is	3.505861	Add
$R - 1 = 0,05$ its Log. is	8.698970	
Num. call'd x 2,7859 its Log. is	0.444969	
Sum	2.649800	Subſt.
Num. $x - 1 = 1,7859$ its Log.	0.251857	
There Remains the Logarithm	2.397943	of 250

That is, 250 l. will be the Rent for Yearly Payments, (*As before.*)

But if the Annuity, or Rent, &c. is to be paid either *Half-yearly*, or *Quarterly*, &c.

Then, *Inſtead of Multiplying the propoſed preſent Worth of the Annuity, with* $R - 1$ (*viz.* with the Ratio of the Rate, as in *Rule* 2.) *you muſt take ſuch a Part of the Log. of* R, *as the Time requires* (*viz.* the $\frac{1}{2}$, the $\frac{1}{4}$ or $\frac{1}{12}$, *&c.*) *and its Number made leſs by* 1, *muſt be the Multiplicator; and then proceed by* Rule 2. *As in the following Examples.*

Example 2. In Half-yearly Payments.

What Annuity, or Rent, to be paid every Half-Year, and to continue 21 *Years, may be Purchaſed for the Sum of* 3244 *l.* 5 *s. at the Rate of* 5 per Cent. per Annum?

In this Queſtion there is given $P=3244,25$ $t=21$ and $R=1,05$ To find U, the Half-yearly Rent.

Thus $R=1,05$ its Log. 0.021189 } Multiply
And $t=$ 21 }
The Product is the Log. 0.444969 of 2,7859 call'd x:

Then $2,7859-1=1,7859$ the Diviſor. *So far the Work is the ſame as for Yearly Payments.*

Next $R=1,05$ the $\frac{1}{2}$ of its Log. is 0.010594 whoſe Number is 1,0247, and $1,0247-1=0,0247$ the Multiplicator inſtead of $0,05=R-1$.

Then $P=3244,25\times 0,0247$ the Product is 80,133 And $80,133\times 2,7859$ the Number call'd x, the Product is 223,2432 for the Dividend.

Then $x-1=1,7859$) 223,2432 ($125=U$.

That is 125 *l.* will be the Half-yearly Annuity, or Rent as was requir'd by the Queſtion.

Or having once found the Number call'd x, and taken 1 from it for the Diviſor; And the Multiplicator uſed inſtead of $R-1$, As before: Then the Multiplication, and Diviſion may be perform'd by *Logarithms*; Thus

$R=$

$$\left.\begin{array}{lr} R = 1{,}05 \text{ its Log.} \times \text{with } t \text{ is} & 0.444969 \\ \text{And } P = 3244{,}25 \text{ its Log. is} & 3.511114 \\ \text{Multiplicator } 0{,}0247 \text{ its Log. is} & 8.392697 \end{array}\right\} \text{Add}$$

$$\left.\begin{array}{lr} \text{Sum} & 2.348780 \\ \text{The Diviſ. is } 1{,}7859 \text{ its Log. is} & 0.251857 \end{array}\right\} \text{Subſtract}$$

There Remains the Logarith. 2.096923 of 125 = U &c.

Example 3. In Quarterly Payments.

What Annuity, or Rent, to be paid every Quarter of a Year, and to continue 31 *Years and a Half, may be Purchaſed for* 1033 l. 3 s. 8 d. *at the Rate of* 7 per Cent. per Annum?

Here is given $P = 1033{,}185$ $t = 31{,}5$ and $R = 1{,}07$
To find U, the Quarterly Payments.

$$\left.\begin{array}{lr} \text{Firſt } R = 1{,}07 \text{ its Log. is} & 0.029384 \\ \text{And } t = & 31{,}5 \end{array}\right\} \text{Multiply}$$

The Product is the Log. 0.925596 of 8,4255 call'd x
And $8{,}4255 - 1 = 7{,}4255$ for the Diviſor.

Again, $R = 1{,}07$ the $\frac{1}{4}$ of its Log. is 0.007346 whoſe Number is 1,01706 and $1{,}01706 - 1 = 0{,}01706$ for the Multiplicator, inſtead of $0{,}05 = R - 1$.

Then $P = 1033{,}185 \times 0{,}01706$ the Product is 17,6261 And $17{,}6261 \times 8{,}4255$ *the Number called* x, the Product will be 148,5087 the Dividend.

Then 7,4255) 148,5087 ($20 = U$; That is 20 *l.* will be the Quarterly Annuity, or Rent as was requir'd.

The ſame may be found by *Logarithms*, as in the laſt Example of Half-yearly Rents; which I ſuppoſe is needleſs to be ſet down.

Caſe 3. When U, P, and R are given; To find t. That is, When any Annuity, with its propoſed Value or Purchaſe, and the Rate of Intereſt *per Cent.* &c. are given; Thence to find how long the Purchaſer ought to enjoy it.

Rule. { *Firſt, Multiply the propoſed Value or preſent Worth of the Annuity, with* R — 1. (*Viz.* with the Ratio of the given Rate of Intereſt) *and Subſtract the Product from the Annuity, the Remainder will be the Diviſor.*

Next, Divide the Annuity by that Diviſor, and find the Log. of the Quotient; Then if that Log. be Divided by the Log. of R *the Amount of* 1 l. *the Quotient will ſhew the Time, for Yearly Payments.*

Example 1. For Yearly Rents.

Suppoſe 3205 l. 5 s. *were Lent upon an Annuity of* 250 l. per Annum; *In what Time would it pay off that Debt, allowing the Creditor* 5 per Cent. per Ann.?

In this Queſtion there is given, $P = 3205,25$ $U = 250$ and $R = 1,05$ To find t.

$$\text{Firſt}\quad\left.\begin{array}{r} P = 3205,25 \\ R - 1 = \underline{0,05} \end{array}\right\}\text{Multiply}$$

And $U = 250 - 160,2625$ leaves 89,7375 the Diviſor.

Next, 89,7375) 250,0000 (2,7859 whoſe Logarithm is 0.444969.

Then $R = 1,05$ its Log. is 0.021189) 0.444969 (21 = t.

That is 21 Years is the Time required in the Queſtion.

The ſame may be perform'd by *Logarithms*; Thus,

$$\left.\begin{array}{lr} \text{If } P = 3205,25 \text{ its Log. is} & 3.505863 \\ \text{And } R - 1 = 0,05 \text{ its Log.} & \underline{8.698970} \end{array}\right\}\text{Add}$$

The Sum is the Logarith. 2.204833 of 160,2625

$$\left.\begin{array}{lr} \text{Next, If } U = 250 \text{ its Log. is} & 2.397940 \\ 250 - 160,2625 = 89,7375 \text{ Log.} & \underline{1.952972} \end{array}\right\}\text{Subſtract}$$

$R = 1,05$ its Log. is 0.021189) 0.444968 (21 = t, *Viz.* 21 Years is the Time, if the Payments are Yearly.

But if the Annuity, or Rent is to be paid, either by Half-yearly, or Quarterly Payments, &c.

Then *Inſtead of Multiplying the propoſed preſent Worth of the Annuity with* $R - 1$ (as before,) *you muſt take ſuch a part of the Logar. of* R *the Amount of* 1 l. *as the Time requires, and the Number anſwering to that part, being made Leſs by* 1, *will be the Multiplicator; and then you may proceed on as before, according to the Rule.*

Example 2. For Half-yearly Rents.

Admit a Man ſhould give 3244 l. 5 s. *for an Annuity, or clear Rent of* 250 l. per Annum; *to be paid Half-yearly* (*viz.* 125 *l.* every Half-Year;) *How long muſt he Enjoy that Annuity, to be allow'd Five* per Cent. &c.?

Here is given $U = 125$, $P = 3244,25$ and $R = 1,05$
To find *t*, the Time or Number of Years

Firſt $R = 1,05$ the $\frac{1}{2}$ of its Log. is 0.010594 whoſe Number is 1,0247 and $1,0247 - 1 = 0,0247$ the Multiplicator inſtead of $0,05 = R - 1$.

Then $P = 3244,25 \times 0,0247 = 80,1329$.

And $U = 125 - 80,1329$ leaves 44,8671 the Diviſ.

Next 44,8671) 125,0000 (2,786 whoſe Logarithm is 0,444981 for the Dividend.

Then $R = 1,05$ its Log. is 0,021189) 0,444981 ($21 = t$.

That is, 21 Years is the Time that is required in the Queſtion.

The ſame perform'd by *Logarithms.*

Suppoſe $R = 1,05$ then the $\frac{1}{2}$ of its Log. 0.010594 whoſe Number is 1,0247 and $1,0247 - 1 = 0,0247$ the Multiplicator inſtead of $R - 1$; As before.

Then $P = 3244,25$ its Log. is 3.511114 } Add
The Multipli. 0,0247 its Log. 8.392697 }
Their Sum is the Logarithm 1.903811 of 80,133

And

Next $U=125$ its Logar. is 2.096910 } Subſ.
And $125-80{,}133=44{,}867$ its Log. 1.651927 }

Then $R=1{,}05$ its Log. is 0.021189) 0.444983 ($21=t$
Viz. 21 Years, *&c.* As before.

Example 3. In Quarterly Payments.

Suppoſe one ſhould give 1033 l. 3 s. 8 d. *for an Annuity of* 80 l. per Annum, *to be paid Quarterly* (*viz.* to have 20 *l.* every Quarter of a Year;) *How Long may be Bnloy it, to be allowed Seven* per Cent. per Annum ?

In this Queſtion there is given $U=20$, $P=1033{,}185$ and $R=1{,}07$ To find t, the Time or Num. of Years.

Thus, $R=1{,}07$ the $\frac{1}{4}$ of its Log. is 0.007346 whoſe Number is 1,01706 and $1{,}01706-1=0{,}01706$ for the Multiplicator, to be uſed inſtead of $0{,}07=R-1$.

Then $P=1033{,}185\times0{,}01706=17{,}6261$; which being taken from $U=20$, Leaves 2,3739 the Diviſor.

Then 2.3739 (20,0000 (8,425 its Log. 0.925570.

Laſtly $R=1{,}07$ its Log. 0.029384) 0.925570($31{,}5=t$
Viz. the Time requir'd will be 31 Years and a Half.

Which may alſo be found by *Logarithms.*

Thus, If $R=1{,}07$ the $\frac{1}{4}$ of its Log is 0.007346 whoſe Number is 1,01706 and $1{,}01706-1$ Leaves 0,01706 for the Multiplicator uſed inſtead of $R-1$.

Then $P=1033{,}185$ its Log. is 3.014181 } Add
Multiplicator 0,01706 its Log. 8.231979 }
Their Sum is the Logarithm 1.246160 of 176262
And if $U=20$ then $20-17{,}6262=2{,}3738$ the Diviſ.

Again $U=20$ its Logar. is 1.301030 } Subſtract
Diviſor 2,3738 its Log. is 0.375444 }
$R=1{,}07$ its Log. is 0.029384)0.925586 ($31{,}5=t$, *&c.*

Caſe 4. By having *U*, *t*, and *P*, given; To find *R*; That is, When any Annuity, or Rent *per Annum*, with the Time of its Continuance, and its propoſed preſent Worth for that Time are given; Thence to find what Rate of Intereſt *per Cent.* &c. is allow'd the Purchaſer.

This Caſe is ſo difficult, that the Ingenious *Michael Dary*, in his Treatiſe of *Intereſt* Epitomiz'd, *Pag.* 7. ſets it down as very troubleſome to be Reſolved, in regard (*ſays he*) that R (*the thing ſought*) cannot be brought to one Side of the Æquation ſolely, and the other Side clear. But I rather think that the true Cauſe which rendred a perfect Solution of this Caſe ſo troubleſome to him (*or indeed impoſſible at that time*) was becauſe the Reſolving of ſuch high Æquations as this Caſe requires, was not in his Time ſo well known as now it is.

For the Æquation, by which the true Value of R may be found, is this,

$$\textit{Viz.}\quad \frac{U}{P} = \frac{U}{P} R^{t} + R^{t} - R^{t+1}$$

Which I Reduce to the following Æquation, as being much better for our preſent *Purpoſe*,

$$\textit{Viz.}\quad U = UR^{t} - PR^{t} \times R - 1.$$

Either of theſe Æquations is more Adfected than that in *Page* 67; and require an Algebraical Solution to diſcover the true Value of *R*; And can be no otherwiſe found here, but only by the ſame manner of Approachment, as before in the 4th *Caſe* of the laſt *Section*: However, in order to render that Approximation as eaſie and ſhort as poſſibly I can, obſerve the following Directions.

Viz.

Firſt, Aſſume ſuch a Number for the Value of R, *as you gueſs may be the Amount of* 1 l. *at the Rate of* *the Intereſt required*, (as before in Page 68, &c.) *Then Multiply the Log. of that ſuppoſed* R *with the Time, and find the Number which belongs to that Product, and Multiply it with the Annuity; Call the Product* z.

Next, Multiply the ſame Number with the propoſed preſent Worth, and Multiply their Product with R — 1, (*viz.* with the Ratio of the ſuppoſed Rate:)

Then Subſtract the laſt Product from the Product call'd z. *If the Remainder be Equal to* (*viz.* be the ſame with) *the Annuity, you have hit right upon the true Value of* R: *But if it be Leſs than the Annuity, then is the Number of* R *taken too Big: And on the contrary, if it be more than the Annuity, that* R *is too Little; and another Tryal muſt be made accordingly.*

Example 1. In Yearly Rents.

Suppoſe there were paid 3205 l. 5 s. *for an Annuity, or a Leaſe of* 250 l. per Annum, *to continne* 21 *Years; What Rate of Intereſt* per Cent. &c. *is allow'd to the Purchaſer?*

In this Queſtion there is given, $U = 250$, $t = 21$, and $P = 3205,25$ To find R the Amount of 1 *l.*

Firſt, I will ſuppoſe R to be 1,06 That is the Amount of 1 *l.* at the Rate of 6 *per Cent.*

Then if $R = 1,06$ its Log. is 0.025306. } Multiply
And $t = 21.$ }

The Product is the Logarith. 0.531426 of 3,3996

Then $U = 250 \times 3,3996$ the Product is 849,9 which is the Number called z.

Next $P = 3205,25 \times 3,3996$ the Product is 10896,5679 And $10896,5679 \times 0,06$ (viz. *with* $R - 1$) the Product will be 653,7941 Then if this Number be Subſtracted from 849,9 the Number called z, there will Remain 196,1059 which is Leſs than 250 the Annuity: And therefore I conclude that the ſuppoſed $R = 1,06$ is too Big.

Again, For a Second Tryal, I will ſuppoſe $R = 1{,}05$

Then if $R = 1{,}05$ its Log. is 0.021189 } Multiply
And $t = 21$ }

The Product will be the Log. 0.444969 of 2.7859.

And $250 \times 2{,}7859 = 696{,}475$ the Number called z.

Next $3205{,}25 \times 2{,}7859 = 8929{,}5$ which being Multiplied with $0{,}05 = R - 1$, the Product is $446{,}475$

Then z, $696{,}475 - 446{,}475 = 250{,}000$ which is the very ſame with the Annuity: Whence I conclude that the true Value of R is 1,05 And $R - 1 = 0{,}05$ will be the Ratio of the Rate of Intereſt ſought.

Then it will be, As 1 : Is to 0,05 :: So is 100 : To 5 the Rate of Intereſt *per Cent.* as was required for Yearly Payments.

Now theſe Trials for finding the true Value of R, may be perform'd by *Logarithms* with a very little trouble. As for Inſtance in this laſt Example,

Wherein $U = 250$, $P = 3205{,}25$ and $t = 21$,
To find R.

Suppoſe $R = 1{,}06$ its Log. is 0.025306 } Multiply
And $t = 21$ }

Sum 0.531426 } Add
Next $U = 250$, its Log. is 2.397940 }

The Sum is the Logarith., 2.929366 of 849,9 the Number called z.

Again $R = 1{,}06$ its Log. x with t, is 0.531426 }
And $P = 3205{,}25$ its Log. is 3.505863 } Add
$R - 1 = 0{,}06$ its Log. is 8.778151 }

The Sum is the Logarrith. 2.815440 of 653,79.

Then z, $849{,}9 - 653{,}79 = 196{,}11$ which is Leſs than 250 the Annuity: Whence it appears that $R = 1{,}06$ is too Big.

2*ly*. Let $R=1{,}05$ its Log. 0.021189 } Multiply
And $t=21$ }

Sum 0.444969 } Add
Next, $U=250$ its Log. 2.397940 }

The Sum is the Logar. 2.842909 of 696,48 call'd z.

Again $P=3205{,}25$ its Log. is 3.505863 }
$R=1{,}05$ its Log. × with t, is 0.444969 } Add
And $R-1=0{,}05$ its Log. is $\overline{8}$.698970 }

Their Sum is the Logarithm 2.649802 of 446,48

Then z, 696,48 — 446,48=250, which is juſt the ſame with the Annuity; Therefore the true Value of R is 1,05 *&c.* As before.

But if the Annuity, or Rent is to be paid, either Half-yearly; or Quarterly, *&c.*

Then { *Inſtead of Multiplying with* R — 1, viz. *with the Ratio of the ſuppoſed Rate of Intereſt* (as above) *you muſt take ſuch a part of the Log. of* R (the aſſumed Amount of 1 *l.*) *as the Time requires, and make its Number the Multiplicator,* &c. *As in the following Examples.*

Example 2. In Half-yearly Payments.

Admit 3244 l. 5 s. *were given for an Annuity, or Leaſe of* 250 l. per Annum, *to be paid Half-yearly,* (*viz.* 125 *l.* every Half-year) *and to continue* 21 *Years; What Rate of Intereſt* per Cent. *is allow'd the Purchaſer.*

In this Example there is given $U=125$, $t=21$, and $P=3244{,}25$ To find R the Amount of 1 *l.*

Firſt, I ſuppoſe $R=1{,}06$ its Log. 0,025306 } Multiply
And $t=21$ }

Their Product is the Logarith. 0.531426 of 3,3996

Then $U=125\times3{,}3996$ the Product is 424,95 call'd z.

Next $P=3244{,}25\times3{,}3996$ the Prod. is 11029,1523

Again, If $R = 1,06$ the ½ of its Log. is 0.012653 whoſe Numb. is 1,02956 and $1,02956 - 1 = 0,02956$ the Multiplicator inſtead of $0,06 = R - 1$.

Then $11029,1523 \times 0,02956 = 326,0217$ which being Subſtracted from 424,95 called *z*; there will Remain 98,9283 which is Leſs than 125, the Half-yearly Annuity; Whence I conclude that the ſuppoſed $R = 1,06$ is taken too Big.

And therefore for a ſecond Tryal I will ſuppoſe $R = 1,05$ and proceed by *Logarithms* only:

Then if $R = 1,05$ its Log. 0.021189 } Multiply
And $t =$ 21 }

Sum 0.444969 } Add
Next $U = 125$ its Log. is 2.096910 }

Their Sum is the Logar. 2.541879 of 348,24 call'd *z*.

Again, if $R = 1,05$ the ½ of its Log. is 0.010594 whoſe Num. is $1,0247 - 1$, Leaves 0,0247 the Multiplicat.

Then $P = 3244,25$ its Log. is 3.511114 }
And $R = 1,05$ its Log. × with t, is 0.444969 } Add
Multiplicator 0,0247 its Log. is $\bar{8}.392697$ }

Their Sum is the Logarith. 2.348780 of 223,24.

Then *z* $348,24 - 223,24 = 125$ which is the very ſame with 125 the Half-yearly Annuity; And therefore I conclude that $R = 1,05$ and $R - 1 = 0,05$ is the Ratio of the Rate of Intereſt ſought.

Conſequently, As 1 : Is to 0,05 :: So is 100 : To 5 the true Rate of Intereſt *per Cent.* &c.

I ſhall cloſe up this *Caſe* with an *Example* of finding the Rate of Intereſt, that is allow'd to the Purchaſers of the *Parliamentary* Annuities; and Reſolve it by *Logarithms* only.

Example 13. Of Quarterly Payments.

There is 1600 l. *paid for an Annuity of* 100 l. per Annum, *to continue* 99 *Years, and to be paid Quarterly*, *viz.* 25 l. *every Quarter of a Year during that Time; 'Tis required to find what Rate of Intereſt* per Cent. *is allow'd the Purchaſer.*

In this Queſtion there is given $U = 25$, $t = 99$, and $P = 1600$; To find R, &c.

Firſt, I will here ſuppoſe the Rate of Intereſt ſought to be 6 *per Cent.* for the Reaſons mentioned in *Pag.* 49.

$$\left.\begin{array}{lr} \text{Then if } R = 1{,}06 \text{ its Log.} & 0.025306 \\ \text{And } t = & 99 \end{array}\right\}\text{Multiply}$$

$$\left.\begin{array}{lr} \text{The Product is} & 2.505294 \\ \text{Next, } U = 25 \text{ its Log.} & 1.397940 \end{array}\right\}\text{Add}$$

The Sum is the Log. 3.903234 of 8002,6 call'd z.

Again, if $R = 1{,}06$ the $\frac{1}{4}$ of its Log. is 0.006326 whoſe Number is 1,01467 Leſs 1, Leaves 0.01467 the Multiplicator.

$$\left.\begin{array}{lr} \text{Then } P = 1600 \text{ its Log. is} & 3.204120 \\ \text{The Log. of } R \text{ Multip. with } t & 2.505294 \\ \text{Multiplicator } 0.01467 \text{ its Log.} & \bar{8}.166430 \end{array}\right\}\text{Add}$$

Their Sum is the Logarith. 3.875844 of 7513,5 which being taken from 8002,6 = z Leaves 489,1 which is More than 25 = U; And therefore the Value of R is taken too Little.

And for a Second Tryal I do Aſſume $R = 1{,}064$

$$\left.\begin{array}{lr} \text{Then if } R = 1{,}064 \text{ its Log.} & 0.026942 \\ \text{And } t = & 99 \end{array}\right\}\text{Multiply}$$

$$\left.\begin{array}{lr} \text{The Product is} & 2.667258 \\ \text{Next } U = 25 \text{ its Log. is} & 1.397940 \end{array}\right\}\text{Add}$$

The Sum is the Logarith. 4.065198 of 11619,8 = z.

Again, the $\frac{1}{4}$ of the Log. of R is 0.006735 its Num. is 1,01563 And $1{,}01563 - 1 = 0.01563$ the Multiplicator.

Then $P = 1600$ its Log. 3.204120 }
Log. of R × with t, is 2.667258 } Add
Multiplic. 0.01563 itsLog. $\bar{8}.193959$ }

Their Sum is the Logar. 4.065337 of 11623,5 which is More than 11619,8 $= z$; And therefore the Value of this $R = 1,064$ is ſome ſmall matter too Big; Conſequently R is ſome Number between 1,06 and 1,064, and is but a very little Leſs than 1,064

Let $R = 1,0638$ its Log. 0.026860 }
And $t =$ 99 } Multiply

Product 2.659140 }
Next $U = 25$ its Log. is 1.397940 } Add

The Sum is the Logar. 4.057080 of 11414,6 $= z$.

Again $R = 1,0638$ the $\frac{1}{4}$ of its Log. is 0,006715 whoſe Num. is 1,01558 Leſs 1 is 0,01558 the *Multiplicator*,

Then $P = 1600$ its Log. is 3.204120 }
Log. of R Multiplied with t 2.659140 } Add
Multiplicat. 0,01558 its Log. $\bar{8}.192567$ }

The Sum is the Logarithm 4.055827 of 1137,7

Then the Num. call'd z, *viz.* $11414,6 - 11377 = 37,6$ which is now a ſmall matter More than $25 = U$.

Conſequently $R = 1,0638$ is a very ſmall matter too Little, which I judge muſt be betwixt 1,0638 and 1,0639; And accordingly I make one other Tryal with aſſuming the Value of $R = 1,063855$ and find it pretty near to the Truth.

Conſequently $R - 1 = 0,063855$ the Ratio of the Rate. And then it will be As $1 : 0,063855 :: 100 : 6,3855$ That is, 6 *l.* 7 *s.* $8\frac{1}{2}$ *d.* will be very near the true Rate of Intereſt *per Cent.* As was required.

And for a Confirmation of the Truth thereof, let us take it for granted that $R = 1,063855$ As above; And let there be given the Quarterly Annuity, or $U = 25$ *l.* the Time or $t = 99$ Years; To find P the preſent Worth (*per Caſe* 1.) which we already know muſt be 1600 *l.* if the Value of *R* be truly found. Then,

If *R* = 1,063855 its Log. is 0.026882 } Multiply
And *t* = 99 }

Their Product is 2.661318 } Add
Next *U* = 25 its Log. is 1.397940 }

The Sum is the Logarithm 4.059258 of 11461,94

From this 11461,94 Subſtract the Quarterly Rent 25, there Remains 11436,94 the Dividend.

Again *R* = 1,063855 the $\frac{1}{4}$ of its Log. is 0.006720 whoſe Num. is 1,015592 and 1,015592 — 1 = 0.015592 the Multiplicator inſtead of 0,063855 = *R* — 1.

The Dividend 11436,94 its Log. is 4.058312

Log. of *R* Multiplied with *t*, is 2.661318 } Add
Multiplicator 0,015592 its Log. is $\overline{8}$.192902 }

From the upper Log. Subſt. this Log. 0.854220 Sum

There Remains the Logarithm 3.204092 of (1599,9 = *P*.

That is, the preſent Worth thus found is 1599 *l.* 18 *s.* which wants but 2 *s.* of 1600 *l.* the true preſent Worth: Whence we may be aſſured, that the Value of *R*, and conſequently the Rate of Intereſt *per Cent.* found as above, is very near the Truth, for Quarterly Payments. But ſuppoſing thoſe Annuities were to be paid but once a Year, then the Rate of Intereſt will be only 6 *l.* 5 *s.* *per Cent.* or very near it, as may be eaſily Tried, either by finding the preſent Worth, as in *Example* 1, *Page* 74. Or by finding the Annuity, as in *Example* 1. *Page* 78, *&c.*

Thus you have a full Solution at large to the Eight Caſes, which include all the Varieties that can be propoſed, either relating to the Arrears; or about the Purchaſing of Annuities, and the Taking of ſuch Leaſes as are either in preſent Poſſeſſion when the Contract is made. Or are to be immediately Entered upon. But for ſuch Queſtions, as relate to Annuities, and Taking ſuch Leaſes, &c. as are in Reverſion, they require a further Conſideration.

Sect. 4. To find the Preſent Worth of Annuities, or Leaſes, *&c.* in *Reverſion* at *Compound Intereſt.*

I have already touch'd upon this Part of the Work *Pag.* 47, &c. as it may be performed at *Simple Intereſt*; But ſhall here handle it more fully in all its uſeful Caſes. For altho' they only depend upon the due Application of the Rules already laid down; yet it will be convenient to ſhew how thoſe Rules are to be truly Applied, according as the particular Caſes require.

Caſe 1. When the Annuity, or Yearly Rent, with the Time of its Continuance in Poſſeſſion, and the Time of the Reverſion, (viz. *the Time it's not to be in Poſſeſſion*) are given; To find its Preſent Worth, at any aſſign'd Rate of Interſt *per Cent.*

The Solution of this Caſe requires Two diſtinct Operations, which muſt be thus perform'd.

First { *Find the Value or Preſent Worth of the propoſed Annuity, or Rent for the given Time of its Continuance, as if it were immediately to be Entered upon.*

Then { *Find what Principal or Sum, being forborn at Intereſt, during the Time of the Reverſion, would Amount to or Raiſe the aforeſaid Value; and that Principal will be the Preſent Worth of the propoſed Annuity,* &c. *in Reverſion.*

Example in Yearly Rents.

There is the Reverſion of a Leaſe, of 175 l. per Ann. *to be Lett for Eleven Years, which are not to commence until after Nine Years are expired: 'Tis required to find the*

the Preſent Worth of that Leaſe, allowing the Rate of Six per Cent. *Intereſt to the Purchaſer.*

Or thus :

There is One has Nine Years to come in a Leaſe of 175 l. per Annum, *and he's deſirous to enlarge his Time Eleven Years more* (*viz.* to Enjoy his Leaſe Twenty Years yet to come ;) *What Sum muſt he give in ready Money for that Purchaſe, to be allow'd the Rate of* 6 per Cent. &c. ?

In the firſt Work of this Example there is given $U=175$, $t=11$, and $R=1{,}06$ To find P the Preſent Worth, As in *Caſe* 1. *Page* 74.

Thus $R=1{,}06$ its Log. is 0.025306 } Multiply
And $t=$ 11 }

The Product is 0.278366 } Add
Again $U=175$ its Log. is 2.243038 }

The Sum is the Logarithm 2.521404 of 332,203

Then from 332,203 Subſtract the Rent 175 and there Remains 157,203 its Log. is 2.196459

The 1ſt Log. × with $t=11$ is 0.278366 } Add
And $R-1=0{,}06$ its Log. is $\bar{8}.778151$ }

From up Log. Subſt. this Log. $\bar{9}.056517$

There Remains the Logarith. 3.139942 of $1380{,}2=P$ the Preſent Worth, ſuppoſing the Rent were to be immediately Entered upon ; but becauſe it is not ſo, therefore, for the Second Part of the Work there is given $A=1380{,}2$, $t=9$, and $R=1{,}06$ To find P the Principal, As in *Caſe* 2. *Page* 54.

Thus $A=1380{,}2$ its Log. is 3.139943 } Subſtract
$R=1{,}06$ its Log. × $9=t$, is 0.227754 }

And there Remains the Log. 2.912189 of $816{,}937=P$

Viz. 816 *l.* 18 *s.* 9 *d.* will be the true Preſent Worth of that Leaſe in Reverſion, as was required, *viz.* for Yearly Rents.

But if the Rents are to be paid, either *Half-yearly*, or *Quarterly*, *&c.* Then the firſt Part of the Work muſt be found as in their reſpective Examples, *viz.* For Half-yearly Payments it muſt be found as in the Example, *Page* 75. And for Quarterly Payments as in the Example, *Page* 76. And then proceed on as above.

Caſe 2. To find what Annuity, or Yearly Rent in Reverſion, may be purchaſed for any propoſed Sum, at any aſſign'd Rate of Intereſt *per Cent.* When the Time the Annuity is not to be Enter'd upon; And the Time of its Continuance are both given.

The Solution of this Caſe is but the Reverſe of the Laſt, and doth alſo require Two diſtinct Operations.

First { *Find what the Sum propoſed to be laid out in the Reverſion, would Amount to, ſuppoſing it were forborn at the given Rate of Intereſt, during the Time that the Annuity is not to be in Poſſeſſion.* }

Then { *Find what Annuity, or Yearly Rent that Amount will Purchaſe, and that will be the Anſwer requir'd.* }

Example in Yearly Rents.

What Annuity, or Yearly Rent, to commence Nine Years hence, and then to continue Eleven Years after, may be Purchaſed for 816 l. 18 s. 9 d. *ready Money; at the Rate of* 6 per Cent. &c.

In the firſt Work of this Queſtion there is given, $P = 816{,}937$ $R = 1{,}06$ and $t = 9$ (*the Time that the Annuity is not to be Enter'd upon;*) To find A; As in *Caſe* 1. *Page* 53.

Thus $R = 1{,}06$ its Log. 0.025306 } Multiply
And $t = 9$ }

The Product is 0.227754 } Add
Next $P = 816{,}937$ its Log. 2.912189 }

The Sum is the Logar. 3.139943 of $1380{,}2 = A$ the

Amount of 816 *l.* 18 *s.* 9 *d.* which we muſt now call P for the ſecond Part of the Work; wherein there is given $P = 1380,2$ $R = 1,06$ and $t = 11$ (*the Time that the Annuity is to be Enjoy'd*) To find U, As in *Caſe* 2. *Page* 77.

Thus $R=1,06$ its Log. is 0.025306 } Multiply
And $t =$ 11 }
The Product is the Log. 0.278366 of 1,8983
And 1,8983 — 1 = 0,8983 is the Diviſor.

Again $R = 1,06$ its Log. is 0.025306 } Multiply
And $t + 1 =$ 12 }
Their Product is the Logar. 0.303672 of 2,0122
And 2,0122 — 1,8983 = 0.1139 the Multiplicator.

Then $P = 1380,2$ its Log. is 3.139943 } Add
Multiplicator 0,1139 its Log. 9.056524 }
Sum 2.196467 } Subſtract
The Diviſor 0,8983 its Log. 9.953421 }
There Remains the Logar. 2.243046 of 175 = U.

That is, 175 *l. per Annum* is the Rent, or Annuity required by the Queſtion.

But if the Rents are to be paid every *Half-Year*, or every *Quarter*, &c. Then the ſecond Part of the Work muſt be perform'd as in their reſpective Examples; *Viz.* For Half-yearly Payments, U muſt be found, as in the Example, *Page* 80. And for Quarterly Payments as in the Example, *Page* 81, &c.

Caſe 3. Any Annuity, or Yearly Rent, and the Sum that's propoſed to be paid for a Leaſe of it, to commence after a certain Term or Number of Years are paſt, being given; Thence to find how Long that Leaſe ought to continue in the Poſſeſſion of the Purchaſer, allowing him any aſſign'd Rate of Intereſt *per Cent.*

This Caſe does alſo require Two ſeparate Operations, *Viz.*

Viz. { *Firſt, To find what Sum the propoſed Money to be laid out in the Reverſion, would Amount to,* &c. *As in the firſt part of the Laſt Caſe.*

Then { *Having the Annuity, with its preſent Worth or Value, and the Rate of Intereſt given; To find the Time,* &c. *As in* Caſe 3. Page 82.

Example in Yearly Rents.

Suppoſe a Debt of 816 l. 18 s. 9 d. *were propoſed to be paid off, by making over a Leaſe of* 175 l. per Annum *in Reverſion, that is not to be Enter'd upon, by the Creditor, until Nine Years are paſt; The Queſtion is, How many Years the Creditor may Enjoy that Leaſe, to have his Debt Cleared; and to be allowed the Rate of* 6 per Cent. per Annum.

For the firſt Work in this Queſtion there is given $P = 816,937$ $R = 1,06$ and $t = 9$ (*the Time before the Leaſe is to commence.*) To find A, which will be found $= 1380,2$ as in the firſt Part of the Laſt Caſe; which we muſt here alſo call P, for the Second Part of the Work.

Then there will be given new $P = 1380,2$ $U = 175$ and $R = 1,06$ To find the $t =$ to the Time which the Leaſe is to be Enjoy'd; As in *Example* 1. *Pag.* 82.

Thus $P = 1380,2$ its Log. is 3.139942 } Add
$R - 1 = 0,06$ its Log. is 8.778151 }

The Sum is the Logarithm 1.918093 of 82,812

Next $U = 175$ its Log. is 2.243038 } Subſtract
$175 - 82,812 = 92,188$ its Log. 1.964674 }

$R = 1,06$ its Log. is 0.025306) 0.278364 ($11 = t$.
Viz. 11 Years will be the Time required by the Queſtion; Suppoſing the Rents to be paid but once a Year.

But if the Rents are to be paid every Half-Year, Then t, the Time the Leaſe is to be Enjoy'd, may be found as in *Example Page* 82. And if Quarterly, then as in *Example Page* 84, &c.

Now theſe Three Caſes include all the uſual (and uſeful) Queſtions that relate to Purchaſing Annuities, or Taking of Leaſes in Reverſion; And altho' I've made choice of ſhewing how to Reſolve each of them by Two ſeparate and diſtinct Operations, (being, as I preſume, the eaſieſt Way to be underſtood) yet they may be Reſolved at one, by a due Reduction of the following Æquation. For

Putting { t = the *Time* the *Leaſe*, &c. is to be Enjoy'd;
T = the *Time* of its *Reverſion*; Then

it will be $UR^t - U = PR^T \times R^t \times R - 1.$

And from hence the Value of R, *viz.* the Rate of Intereſt may alſo be found; which I take to be a Work more of Curioſity, than of any great Uſe: For the Rate of Intereſt is always ſuppoſed to be known, it being one of the main or chief Conſiderations in all Bargains about Taking, or Letting of Leaſes, &c. And therefore the finding of R in the Caſe of Reverſions, (or indeed in the Two 4th Caſes of the Two laſt Sections) is rather for a Proof of the Work, and to Compleat the Rounds, than for any thing elſe. However, if any one will (out of his Curioſity) give himſelf the Trouble of doing it, he may proceed by ſuch Approximation as in the aforeſaid Sections, having a due regard to the different Values of t, and T. But for Brevity ſake I ſhall omit giving Examples thereof.

CHAP.

CHAP. V.

Of *Purchaſing* Free-hold or Real Eſtates, and how to *Eſtimate* the Value of Annuities, and Leaſes for *Lives*, &c.

ALL *Free-hold* or *Real Eſtates*, are ſuppoſed to be Purchaſed (or Bought) to continue for Ever; *Viz.* without the Conſideration of any limited Time: Therefore the Buſineſs of Computing the true Value or Worth of ſuch Eſtates (*according to Art*) is grounded upon a Rank, or Series, of Geometrical Proportionals, continually Decreaſing *ad Infinitum*; (*As may be ſeen in the aforeſaid Young Mathematician's Guide*, Pag. 276.) from whence the following *Rules* are deduced.

Sect. 1. Of *Purchaſing* Free-hold Eſtates.

I ſhall here make uſe of the ſame Letters to denote the ſeveral Parts of the Queſtion, as in the laſt Chapter, ſave only that *t* the Time to limit the Number of Payments, is not concern'd.

Viz. $U =$ The propoſed *Rent* of an *Eſtate*.
$P =$ The *Purchaſe*, or *Worth* of that *Eſtate*.
$R =$ The *Amount* of 1 *l.* at any Rate of *Intereſt*.

Then it will be $PR - P = U$.

This Æquation admits of Three different Caſes.

Caſe 1. The Annual Rent of any Free-hold Eſtate being known; To find what it's Worth in Ready Money, allowing the Purchaſer any aſſign'd Rate of Intereſt *per Cent.*

Rule. *Divide the propoſed Rent by the Ratio of the given Rate of Intereſt* (*viz.* by $R - 1$) *and the Quotient will ſhew what the Eſtate is Worth at that Rate of Intereſt.*

Example 1.

Suppoſe a Free-hold Eſtate of 250 l. *Yearly Rent, were to be Sold; What is it Worth, allowing the Buyer the Rate of* 6 per Cent. *Compound Intereſt for his Money?*

Here is given $U = 250$, $R = 1{,}06$ To find P.

Thus $R - 1 = 0{,}06$) 250,00 ($4166{,}6667 = P$.

That is, 4166 *l.* 13 *s.* 4 *d.* is the Preſent Worth of that Eſtate; which is 16 Years and 8 Months Value, ſuppoſing the Rents to be paid but once a Year.

The ſame may be found by *Logarithms*,

Thus $U = 250$ its Log. 2.397940 } Subſtract
And $R - 1 = 0{,}06$ its Log 8.778151 }

There Remains the Log. 3.619789 of $4166{,}66 = P$.
Viz. 4166 *l.* 13 *s.* 4 *d.* *&c.* As above.

Note, *Here it is ſuppoſed that the Purchaſer is to Enter immediately into Poſſeſſion of the Eſtate: But if it be an Eſtate in Reverſion, that is, not to be Entered upon until after ſome Term of Years are paſt; Then you muſt Compute its preſent Worth, as in* Caſe 1. Pag. 93. *And ſo on in the following Caſes, compared with their reſpective Caſes in* Sect 4. *of the Laſt* Chapter.

If the Rents are to be paid either Half-yearly, or Quarterly, as moſt generally they are;

Then { *Inſtead of Dividing the propoſed Rent by* $R - 1$, *as above, you muſt take ſuch a part of the Log. of* R *the Amount of* 1 l. *as the Times of Payment requires; and the Number anſwering to that part, being made Leſs by* 1, *muſt be made the Diviſor inſtead of* $R - 1$.

Example in Half-yearly Rents.

Suppoſe the aforeſaid 250 l. per Annum *were to be paid by Half-yearly Rents* (*viz.* 125 *l.* every Half-Year) *What*

What would that Eſtate be then Worth, at the ſame Rate of Intereſt, viz. 6 per Cent. &c.?

In this Example there is given $U=125$ and $R=1{,}06$ To find P. Thus,

Firſt $R=1{,}06$ the $\frac{1}{2}$ of its Log. is 0.012653 whoſe Number is 1,029563 and $1{,}029563-1=0{,}029563$ for the New Diviſor inſtead of $R-1$.

Then $U=125$ its Log. is 2.096910 } Subſtract
Diviſor 0,029563 its Log. $\bar{8}.470748$ }

There Remains the Logar. 3.626162 of $4228{,}26=P$.

So that 4228 *l.* 5 *s.* is here the preſent Worth or Value of the ſame Eſtate, which is 61 *l.* 11 *s.* 4 *d.* More than its Worth for Yearly Rents.

Example 3. For Quarterly Rents.

Let it be required to find what the ſame Eſtate (viz. 250 l. *per Annum*) *is Worth, when the Rents are to be paid Quarterly* (*viz.* 62 *l.* 10 *s.* every Quarter of a Year) *at* 6 per Cent. *Intereſt*, &c. *As before.*

Here is given $U=62{,}5$ $R=1{,}06$ To find P.

Thus, $R=1{,}06$ the $\frac{1}{4}$ of its Log. is 0,006326 whoſe Num is 1,014674 and $1{,}014674-1=0{,}014674$ the Diviſor inſtead of $R-1$.

Then $U=62{,}5$ its Log. is 1.795880 } Subſtract
Diviſ. 0,014674 its Log. $\bar{8}.166548$ }

There Remains the Log. 3.629332 of $4259{,}24=P$

Viz. 4259 *l.* 4 *s.* 9½ *d.* is now the Value, or preſent Worth of the ſame Eſtate; which is ſomething above 17 Years Value.

Now from hence it plainly appears, that, although there is no ſuch thing as a limited Time conſidered in the Purchaſing of Free-hold Eſtates; yet there is, and ought to be, a due Reſpect had to the Times of their Rents being paid; for according to that, their Values are More, or Leſs Worth.

Caſe 2. When any Sum of Money is propoſed to be laid out in a Free-hold Eſtate; To find what Annual Rent that Sum will Purchaſe, at any given Rate of Intereſt *per Cent.* &c.

Rule. *Multiply the propoſed Sum to be laid out in the Eſtate, with* R — 1; viz. *with the Ratio of the given Rate of Intereſt* per Cent. *and the Product will ſhew the Yearly Rent required.*

Example in Yearly Rents.

Suppoſe 3560 l. *were propoſed to be laid out in the Purchaſe of a Free-hold Eſtate; What Annual Rent would it Buy, allowing the Purchaſer but Four* per Cent. per Annum *Intereſt.*

In this Example there is given $P=3560$ $R=1{,}04$
To find U the Yearly Rent.

$$\left.\begin{array}{l} \text{Thus } P = 3560 \\ \text{And } R-1 = 0{,}04 \end{array}\right\}\text{Multiply.}$$

Product is $142{,}4 = 142$ *l.* 8 *s.* $= U$ the Yearly Rent required.

Or the ſame may be found by *Logarithms.*

$$\left.\begin{array}{l} \text{Thus } P = 3560 \text{ its Log. is } 3.551450 \\ \quad R-1 = 0{,}04 \text{ its Log. is } 8.602060 \end{array}\right\}\text{Add}$$

The Sum is the Logarith. 2.153510 of $142{,}4 = U$

Viz. 142 *l.* 8 *s.* will be the Rent required, if it be paid but once a Year; which is juſt 25 Years Value.

But if the Rent muſt be paid *Half-yearly*, or *Quarterly*, &c.

Then *Inſtead of Multiplying the Sum propoſed to be Laid out in the Purchaſe, with* R — 1 (as above) *you muſt take ſuch a part of the Log. of* R (*viz.* of the Amount of 1 *l.*) *as the Times of Payment require, and the Number anſwering to that part being made leſs by* 1, *muſt be the Multiplicator inſtead of* R — 1.

Take

Take the ſame Example in Half-yearly Rents.

That is, Let there be given $P = 3560$ and $R = 1,04$ To find U the Half-yearly Rent.

Firſt $R = 1,04$ the ½ of its Log. is 0.008516 whoſe Number is 1,0198 Leſs 1, is 0.0198 the Multiplicat.

Then 3560 × 0,0198 = 70,488 *viz.* 70 *l.* 9 *s.* 9 *d.* = U the Half-yearly Rent.

Or, having found the New Multiplicator, as above;

$$\left.\begin{array}{ll}\text{Then } P = 3560 \text{ its Log. is} & 3.551450 \\ \text{Multiplicat. } 0,0198 \text{ its Log.} & \underline{8.296665}\end{array}\right\}\text{Add}$$

The Sum is the Logarith. 1.848115 of 70,488 = U;
Viz. 70 *l.* 9 *s.* 9 *d.* is the Half-yearly Rent, As before.

Example 3. For Quarterly Rents.

Suppoſe the ſame Sum of 3560 l. *were propoſed to be Laid out in the Purchaſe of a Free-hold Eſtate; and it were required to find what Quarterly Rent it would Buy, at the Rate of Four* per Cent. &c.

Here is alſo given $P = 3560$ and $R = 1,04$ To find U the Quarterly Rent. Thus,

Firſt $R = 1,04$ the ¼ of its Log. is 0.004258 whoſe Number is 1,00986 and 1,00986 — 1 = 0,00986 the New Multiplicator.

Then 3560 × 0,00986 gives 35,1 *viz.* 35 *l.* 2 *s.* = U the Rent to be paid every Quarter; which is Leſs than the ¼ of the Annual Rent by 10 *s.* *per* Quarter.

Whence it plainly appears, that the Leſs the Intervals of the Times are, betwixt the Payments of the Rents, the More valuable the Purchaſe is; & *vice verſâ*,

Caſe 3. The Annual Rent of any Free-hold Eſtate, and the Sum that is paid for it, being known; To find what Rate of Intereſt *per Cent.* is allow'd to the Purchaſer.

Rule. { *Divide the Annual Rent by the Sum that is paid for the Purchaſe, and the Quotient will ſhew the Ratio of the Rate of Intereſt* per Cent.

Example.

Suppoſe a Free-hold Eſtate of 250 l. per Annum, *Coſt* 4166 l. 13 s. 4 d. *What Rate of Intereſt* per Cent. *is allow'd the Purchaſer?*

In this Example there is given $U=250$ $P=4166,667$ To find R, or rather $R-1$, the Ratio of the Rate. Thus 4166,667) 256,000 ($0.06=R-1$. Then it will be As 1 : Is to 0,06 :: So is 100 : To 6, the Rate of Intereſt *per Cent.* As was requir'd.

Or by *Logarithms*; Thus,

Firſt $U=250$ its Log. is 2.397940 } Subſtract
And $R=4166,667$ its Log. 3.619789 }

The Difference is the Log. $\overline{8}$.778151 of $0,06=R-1$
Or to the laſt Log. Add this 2 000000 of 100.

The Sum will be the Log. 0.778151 of 6 the Rate of Intereſt *per Cent.* As before.

Thus far concerning ſuch Queſtions that are about *Intereſt*, *Annuities*, or *Leaſes*, &c. as are limited by any aſſign'd Time, and Real Eſtates that are Purchaſed to Continue for Ever; And 'tis only ſuch that can be Computed by any Certain Rules: However, it may not (I preſume) be unacceptable to the Reader if I proceed a little further, and inſert a brief Account of ſuch Helps to Eſtimate the Values of Annuities, or Leaſes for Lives, as are propoſed by Two very Ingenious Perſons.

Section 2. How to *Eſtimate* the *Value* of Annuities, or Leaſes for *Lives*.

The Way generally uſed in Buying of Annuities, or Letting of Leaſes for Lives, is only by an imaginary

Valuation, grounded upon Cuſtom, and not upon any Conſideration that is had to the Age of the Perſons whoſe Lives are to be inſerted in the Leaſe, *&c.* 'Tis true indeed, that there can be no certain Rules preſcrib'd for their true Valuation, becauſe the Lives of all Mankind are uncertain; and 'tis poſſible, and daily ſeen, that a Young Man may Die before one of a greater Age: But yet there is a greater Probability of a Young Man's Living longer than an Old one; And not only ſo, but there's a Proportional Likelihood of the Length of Mens Lives, according to their different Degrees of Age; The which being duly conſidered, muſt needs be found of good Uſe in Eſtimating the Values of Annuities or Leaſes for Lives, much better than by a meer Gueſſing only, as uſual: And that ſuch a Proportional Likelihood is worth the Conſideration, will appear from what follows.

1. Sir *William Petty*, in a Diſcourſe of his, made before the Royal Society (*Anno* 1674.) concerning the Uſe of **Duplicate Proportion**, doth amongſt other things, apply it to the Life of Man, and its Duration, Thus,

It is found by Experience (*ſaith he*) that there are more Perſons Living of between 16 and 26 Years old, than of any other Age or Decad of Years in the whole Life of Man (*which* David *and Experience ſay to be between* 70 *and* 80 *Years*) The Reaſons whereof are not abſtruſe; *viz.* becauſe thoſe of 16 have paſſed the Danger of *Teeth*, *Convulſions*, *Worms*, *Rickets*, *Meazles*, and *Small-Pox* for the moſt part: And that thoſe of 26 are ſcarce come to the *Gout*, *Stone*, *Dropſie*, *Palſies*, *Lethargies*, *Apoplexies*, and other Infirmities of Old Age. Now whether theſe be ſufficient Reaſons, is not the preſent Enquiry; but taking the aforemention'd Aſſertions to be true, I ſay, that the Roots of every Number of Men's Ages under 16 (*whoſe Root is* 4) compared with the ſaid Number 4, doth ſhew the Proportion of the Likelihood of ſuch Men's reaching to 70 (*or* 80) Years of Age.

As for Example; 'Tis Four times more likely, that one of 16 Years old ſhould live to 70, than a new-born Babe. 'Tis Three times more likely, that One of 9 Years old ſhould attain the ſaid Age of 70, than the ſaid Infant. Moreover, 'tis Twice as likely, that One of 16 ſhould reach that Age, as that One of 4 Years old ſhould do it; and One Third more likely, than for One of 9.

On the other Hand, 'Tis Five to Four, that One of 25 Years old will Die before One of 16; and Six to Five, that One of 36 will Die before One of 25; and Three to Two, that the ſame Perſon of 36 ſhall Die before him of 16; And ſo forward, according to the Roots of any other Year of the declining Age, compared with a Number between 4 and 5 (viz. *with* 4,58) which is the Root of 21, the moſt hopeful Year for Longævity, as the Mean between 16 and 26; and is the Year of Perfection, according to the Sence of *Our Law*, and the Age for whoſe Life a *Leaſe* is moſt valuable.

To prove all which, I can produce the Accompts of every Man, Woman, and Child within a certain Pariſh of 330 Souls; All which particular Ages being Added together, and the Sum Divided by the whole Number of Souls, made the *Quotient* between 15 and 16; which I call the Age of that Pariſh, or Numerous *Index* of Longævity there.

Thus you have a Learned Gentleman's Opinion concerning the Likelihood of the Length of Men's Lives, according to the Rules of *Duplicate Proportion*; which was a very ingenious Thought of His: But I muſt beg Pardon, that I cannot agree with Him in that Part of it, which aſſerts, That 21 Years is the Age for whoſe Life a Leaſe is the moſt valuable: For although 'tis true, that according to our Law, a Man is ſaid to be then at his Perfect or Full Age, as to the Enjoyment of an Eſtate, or Managing his Affairs without a Guardian, *&c.* Yet I ſhould rather adhere to the Cloſe of His Diſcourſe, wherein He ſays, That He found, that if the Sum of all the Ages of the 330 Souls (*in a certain Pariſh*) being Divided by their Numbers, made the Quotient between 15 and 16. Whence I take 16 to be the Age, for whoſe Life a Leaſe is the moſt valuable; And upon that Suppoſition I have Calculated the following Table, according to the aforeſaid Rules of *Duplicate Proportion.*

Ages	*Year's Purch.*	*Ages*	*Year's Purch.*	*Ages*	*Year's Purch.*
1	2,50	26	7,20	51	5,04
6	5,51	31	6,46	56	4,81
11	7,46	36	6,00	61	4,60
16	9,00	41	5,62	66	4,43
21	7,85	46	5,31	71	4,27

This Table ſhews, by Inſpection, the Value of every Five Years of any ſingle Life, from the Birth to 71 Years old. Suppoſing that any Annuity, or Leaſe, &c. is really Worth Nine Year's Purchaſe for One Life; which is according to the Rate that the Annuities ſettled by a late Act of Parliament for Lives, were valued at, and from thence the reſt are computed.

As for Inſtance; An Annuity, or Leaſe of 100 *l.* *per Ann.* is by this Table Worth 900 *l.* (*viz.* 9 × 100) for the Life of One that is about 16 Years old; and 'tis Worth but 600 *l.* for the Life of One that is 36 Years old, and but 250 *l.* for the Life of a Child of a Year old: And ſo on for any other propoſed Annuity, or Leaſe, and Age of the Perſon for whoſe Life it is to be Purchaſed: Which is ſo eaſie to apprehend, that I need ſay no more of its Uſes, but proceed to the next.

2. The Ingenious and Great Mathematician, Mr. *Edmund Halley*, *Savilian Profeſſor* of *Geometry* in the *Univerſity* of *Oxford*, hath (*in the Philoſophical Tranſactions*, *Numb.* 196.) given us a moſt Excellent *Eſſay* to Eſtimate the Degrees of the *Mortality* of Mankind, which he deduced with a great deal of Art and Labour, from

curious *Tables* of the *Births* and *Funerals* that were in *Breſlaw*, the Capital City of the Province of *Sileſia* in *Germany*, for Five Years ſucceſſively, *viz*. from the 1687, to 1691 incluſive, drawn up Monthly by one *Doctor Newman* of that City, and communicated to the *Royal Society* here; from whence Mr. *Halley* hath form'd the following *Table*; which is, without doubt, very carefully and exactly done, and does give a more perfect Account of the State and Condition of Mankind in reſpect of their *Chances* of *Mortality* at all *Ages*, and conſequently how to Eſtimate the *Values* of *Annuities*, or *Leaſes* for Lives, better than by any other Method ever propoſed before it. The *Table* does ſhew the *Number* of *Perſons* that were Living in their reſpective *Ages Current* annexed thereto, as follows.

Age Curr.	*Perſons Living.*	*Age Curr*	*Perſons Living.*	*Age Curr.*	*Perſons Living.*	*Age Curr.*	*Perſons Living.*
1	1000	22	586	43	417	64	202
2	855	23	579	44	407	65	192
3	798	24	573	45	397	66	182
4	760	25	567	46	387	67	172
5	732	26	560	47	377	68	162
6	710	27	553	48	367	69	152
7	692	28	546	49	357	70	142
8	680	29	539	50	346	71	131
9	670	30	531	51	335	72	120
10	661	31	523	52	324	73	109
11	653	32	515	53	313	74	98
12	646	33	507	54	302	75	88
13	640	34	499	55	292	76	78
14	634	35	490	56	282	77	68
15	628	36	481	57	272	78	58
16	622	37	472	58	262	79	49
17	616	38	463	59	252	80	41
18	610	39	454	60	242	81	34
19	604	40	445	61	232	82	28
20	598	41	436	62	222	83	23
21	592	42	427	63	212	84	20

This *Table* may be Applied to very many Uſes, but I ſhall only inſert what may ſuffice for the preſent Purpoſe.

1. The Firſt Uſe is to ſhew the Differing Degrees of *Mortality*, or rather *Vitality* in all *Ages*; For if the Number of Perſons of any Age Remaining after One Year, be *Divided* by the *Difference* between that and the *Number* of the *Age* propoſed, it ſhews the *Odds* that there is, that a Perſon of that Age does not *Die* in One Year.

As for Inſtance, a Perſon of 25 *Years* of *Age* has the the *Odds* of 560 to 7 (*viz.* 80 to 1) that he does not *Die* in a Year: Becauſe that of 567 Perſons Living of 25 Years of Age, there do *Die* no more than 7 in a Year, Leaving 560 of 26 Years Old: Or,

2. If it be required to find the *Odds*, that any Perſon does not Die before he attain to any propoſed *Age*. Then,

Take the *Number* of the Remaining Perſons of the *Age* propoſed, and *Divide* it by the *Difference* between it and the *Number* of thoſe of the *Age* of the Party propoſed; and that ſhews the *Odds* there is between the *Chances* of the Parties Living, or Dying.

As for Inſtances; What is the *Odds* that a Man of 40 may Live 7 Years? Take the *Number* of Perſons of 47 Years, which in the *Table* is 377, and *Subſtract* it from the *Number* of Perſons of 40 Years, which is 445 and the *Difference* is 68, *viz.* $445 - 377 = 68$ which ſhews that the Perſons Dying in the 7 Years are 68, and that 'tis 377 to 68, or $5\frac{1}{2}$ to 1, that a Man of 40 does Live 7 Years: And the like for any other Number of Years.

3. And if it be required to find at what *Number* of Years, it is an even Lay that a Perſon of any propoſed *Age* ſhall *Die*, this Table readily performs it: For if the *Number* of Perſons *Living* of the *Age* propoſed be halv'd, it will be found by the *Table* at what Year the ſaid *Number* is Reduc'd to Half by *Mortality*; and that is the *Age*, to which it's an even Wager, that a Perſon

ſon of the *Age* propoſed ſhall arrive to before he Die.

As for Inſtance, a Perſon of 30 Years of *Age* is propoſed; the *Number* of that Age is 531, the Half of it is 265; which *Number* I find to be between 57 and 58 Years: So that a Man of 30 may reaſonably expect to Live between 27 and 28 Years.

4. By what has been ſaid, the *Price* of *Inſurance* upon *Lives* ought to be Regulated, and the *Difference* is diſcovered between the *Price* of *Inſuring* the *Life* of a Man of 20 and One of 50 Years *Old*.

For Example; It being 100 to 1, that a Man of 20 *Dies* not in a Year, and but 38 to 1, for a Man of 50 Years of Age.

5. And upon theſe *Proportions* depend the *Valuation* of *Annutties* for Lives: For it's plain, that the Purchaſer ought to pay only for ſuch a Part of the *Annuity*, as he hath Chances that he is Living; and this ought to be Computed Yearly, and the Sum of all thoſe Yearly *Values* being *Added* together, will be the *Value* of the *Annuity* for the Life of the Perſon propoſed. Now the preſent *Value* of Money payable after any Term of Years, may be eaſily Computed by *Caſe* 2. *Page* 54. at any given Rate of Intereſt.

Then

As the Number of Perſons Living after that Term of Years, to the Number Dead; So are the Odds that any one Perſon is Alive or Dead.

And by conſequence, As the Sum of both, or the Number of Perſons Living of the Age firſt propoſed: To the Number Remaining after ſo many Years (both being given by the *Table*) : : *So is the preſent Value of the Yearly Sum payable after the Term propoſed: To the Sum which ought to be paid for the Chance the Perſon has to Enjoy ſuch an Annuity ſo many Years. And this being repeated for every Year of the Perſon's Life, the Sum of all the preſent Values of thoſe Chances is the true Value of the Annuity.*

Now becauſe the Work of theſe Proportions, is ſomewhat troubleſome to perform; the *Ingenious Author* hath been ſo kind as to take the Pains (*which was not a little*) to Calculate the following *Table*; which ſhews the *Value* of *Annuities*, or *Leaſes*, &c. (*at the Rate of* 6 per Cent.) for every Fifth Year of Age to the Seventieth.

Ages	*Year's Purch.*	*Ages*	*Year's Purch.*	*Ages*	*Year's Purch.*
1	10,28	25	12,27	50	9,21
5	13,40	30	11,72	55	8,51
10	13,44	35	11,12	60	7,60
15	13,33	40	10,57	65	6,54
20	12,78	45	9,91	70	5,32

This *Table* being of the ſame Nature with that in *Pag.* 107, there needs no other Explanation or Example to ſhew its Uſe, than what has been already ſaid about that *Table*: Only here I muſt again beg leave to give my Opinion about the *Difference* of the *Proportions* in the Two *Tables*; which is, that as the *Table* in *Page* 107, may not be thought a ſufficient Guide to be depended upon in Eſtimating the Length of Men's Lives, &c. becauſe its only deduced from the bare Rules of Art, *viz.* that of *Duplicate Proportion*; ſo on the other Hand, I doubt the Eſtimates of the Value of any Annuity taken from this *Table*, will be found too great in this Country, (viz. *in* England) which I much fear hath not ſo Good or *Salubrious* an *Air*, as that at the City of *Breſlaw*, from whence theſe *Calculations* are drawn: But if in Imitation hereof the Curious in other *Cities* and *Large Towns* would attempt ſomething of the ſame Nature; Then without all doubt this Method of Eſtimating the Probability of the *Length* of Mens's Lives, would prove the Beſt, and become more univerſally Uſeful than can be expected from this one ſingle Inſtance, more eſpecially if ſuch Obſervations were continued for any conſiderable Time, as 20, or 30 Years ſucceſſively. And then it would be well worth the Time and Pains to Calculate proper *Tables*, of the *Value* of *Annuities*, or *Leaſes*, &c. both for Two, and for Three Lives, according to the following Rules; which are deduced from the former *Table*.

And Firſt, Two Lives being propoſed, their Values may be thus found.

If the *Number* of *Chances* of each ſingle Life, found in the *Table*, be Multiplied together, the *Product* is the *Chances* of thoſe Two Lives; And after any certain Term of Years, the *Product* of the Two Remaining Sums, will be the *Chances* that both the Perſons are Living: And the *Product* of the *Differences*, being the *Numbers* of the *Dead* of both *Ages*, will be the *Chances* that both the Perſons are *Dead*.

Then *As the Product of the Two Numbers in the Table for the Two Ages propoſed : Is to the Difference between that Product and the Product of the Two Numbers of the Perſons Deceaſed in any given Space of Time :: So is the Value of a Sum of Money to be paid after that Time : To the Value thereof under the Contingency of Mortality.*

And if Three Lives are propoſed, to find the Value of an Annuity during the Continuance of thoſe Lives;

Then *As the Product of the continual Multiplication of the Three Numbers in the Table, anſwering to the Ages propoſed : Is to the Difference of that Product, and the Product of the Three Numbers of the Deceaſed of thoſe Ages in any given Term of Years :: So is the preſent Value of a Sum of Money, to be paid certainly after ſo many Years : To the preſent Value of the ſame Sum to be paid, provided One of thoſe Three Perſons be Living at the Expiration of that Term of Years.*

Theſe Proportions being Yearly repeated, the Sum of all thoſe preſent Values will be the *Value* of the *Annuity* granted for thoſe *Lives*.

Theſe Rules are Explain'd at Large by their Author, both *Algebraically* by Letters, and by *Geometrical Figures*: And he alſo proceeds on (*by the ſame Method*) to Compute the preſent *Value* of the *Reverſion* of any *Annnity*, or Leaſe, either of One Life after another; or after Two Lives, &c. The which being not only too long a Diſcourſe to be inſerted in this ſmall *Treatiſe*, but alſo too Difficult a Piece a Work for any Ordinary Arithmetician to undertake, I have therefore omitted it, and refer thoſe that are Curious, and deſire further Satisfaction therein, to the aforemention'd *Philoſ. Tranſ. Numb.* 196. and ſhall only Add this ſerious Obſervation, *Viz.* How unjuſtly we generally Repine at the Shortneſs of our Lives, and think ourſelves Wrong'd if we attain not to Old Age; Whereas it appears, that the One Half of thoſe that are *Born*, do *Die* in *Seventeen Year's Time*. For by the aforeſaid Bills of *Mortality* at *Breſlaw* it was found, that 1238 were in that Time Reduced to 616. So that inſtead of *Murmuring* at what we call a *Short Life*, we ought with Patience and Unconcern to ſubmit to that Diſſolution which is the neceſſary Condition of our periſhable Materials, and of our nice and frail Structure and Compoſition; And to account it as a great *Bleſſing* that we have ſurvived, (perhaps by many Years) that *Period* of Life, whereat the One Half of the whole Race of Mankind does not Arrive.

Having now gone through all the General Caſes of Intereſt, and Annuities, &c. I deſign'd to have concluded here; But becauſe the Buſineſs of *Rebate* or *Diſcompt* of Money paid before the Time it becomes Due, comes often into Practice upon ſeveral Occaſions, and being but juſt touch'd upon in Page 24 and 54, altho' even what is there done, being duly conſider'd, might be ſufficient; yet leſt I ſhould be thought too ſhort or remiſs in ſo uſeful a Part of Intereſt as that is: It may be convenient to proceed a little further, and lay down particular Rules for that Purpoſe.

CHAP. VI.

To find the Rebate or Diſcompt of any propoſed Sum, &c. and the true Equated Time of ſeveral Payments; either at *Simple*, or *Compound Intereſt*.

Sect. 1. *To find the true Diſcompt of any Sum, at any Rate of Simple Intereſt.*

Suppoſe $\begin{cases} S = \text{the } \textit{Sum} \text{ propoſed to be Diſcompted for;} \\ T = \text{the } \textit{Interval} \text{ of } \textit{Time} \text{ it becomes Due;} \\ R = \text{the } \textit{Ratio} \text{ of the } \textit{Rate} \text{ of } \textit{Intereſt per Cent.} \end{cases}$

And D = the *Diſcompt* or *Rebate* ſought.

Then it will be $\left\{ \frac{TRS}{TR+1} = D. \right.$

Which in Words gives this following Rule.

Rule. *First Multiply the given Time with the Ratio of the Rate of Intereſt, and to their Product Add* 1, *that Sum will be the Diviſor.*
Next Multiply the firſt Product and the Sum that's to be Diſcompted for, and that Product will be the Dividend; The Quotient ariſing from thence will ſhew the Diſcompt requir'd.

Example.

Let it be required to find what Diſcompt ought to be allowed for 3560 l. *if it be paid* 273 *Days before it becomes Due, at the Rate of Six* per Cent. per Annum *Simple Intereſt?*

Here is given $S = 3560$, $T = 0{,}74794$ (*found by the Table in* Pag. 29.) And $R = 0{,}06$ To find D.

First $0{,}74794 \times 0{,}06 = 0{,}044876$

And $0{,}044876 + 1$ is $1{,}044876$ for the Diviſor.

Next $0{,}044876 \times 3560 = 159{,}75856$ the Dividend.

Then $1{,}044876\ )\ 159{,}75856\ (\ 152{,}897 = D$.

That is 152 *l.* 18 *s.* *ferè*, will be the Diſcompt required.

The ſame perform'd by *Logarithms*.

Firſt, $T=$ 0,74794 its Log. 9.873867 }
And $R=$ 0,06 its Log. is 8.778151 } Add.

The Sum is the Log. 8.652018 of 0,044876
Next $S=$ 3560 its Log. is 3.551450 Add to the laſt

Sum 2.203468 }
0,044876+1=1,044876 Log. 0.019066 } Subſtract

There Remains the Logar. 2.184402 of 152,897 That is 152 *l.* 18 *s.* As above, which being Subſtracted from 3560 *l.* the propoſed Sum, there will Remain 3407 *l.* 2 *s.* the Sum to be paid in ready Money; as may be eaſily proved by making it a Principal, and then finding what it would Amount to in 273 Days at 6 *per Cent.* which by *Caſe* 1. *Page* 22. will be juſt 3560 *l.* Conſequently the Diſcompt is truly found.

Secondly, To find the true Rebate or Diſcompt of any Sum, at any given Rate of Compound Intereſt.

Let us ſuppoſe S, t, and D, to repreſent the ſame Parts of the Queſtion as before; And R, to denote the Amount of 1 *l.* &c. as in *Page* 53.

$$\text{Then will } \left\{ \frac{SR^t - S}{R^t} \right. = D.$$

And from this Æquation is deduced the following Rule.

Rule. *Multiply the Logarithm of* R *the Amount of* 1 l. *at the given* Rate *of Intereſt, with the Time, and the Number which belongs to their Product will be the Diviſor.*

From that Number Subſtract 1, *then Multiply the Remainder with the propoſed Sum that's to be Diſcompted for, and that Product will be the Dividend; the Quotient ariſing from thence will ſhew the Diſcompt required.*

Example.

Suppoſe it were required to find what Diſcompt or Rebate muſt be allow'd for the Payment of 956 l. 10 s.

Nine Months (*viz.* ¾ of a Year) *before it becomes Due, at the Rate of Five and a Half* per Cent. *Compound Intereſt.*

In this Queſtion there is given $S = 956,5$ $t = 0,75$ and $R = 1,055$ To find D.

Firſt $R = 1,055$ its Log. is 0.023252 } Multiply
And $t = 0,75$ }

The Product is the Log. 0.017439 of 1,04097 the Diviſor: And $1,04097 - 1 = 0,04097$ will be the Multiplicator: And $956,5 \times 0,04097 = 39,1878$ the Dividend. Then 1,04097) 39,1878 ($37,645 = D$; *viz.* 37 *l.* 12 *s.* 10¾ *d.* is the Diſcompt required.

Or, having firſt found the Diviſor, and by it the Multiplicator, As above; Then the reſt of the Work is very eaſily perform'd by *Logarithms.*

Thus $S = 956,5$ its Logar. is 2.980685 } Add
Multiplicator 0,04097 its Log. 8.612466 }

Sum 1.593151 } Subſtract
Diviſor is 1,04097 its Log. is 0.017439 }

There Remains the Logarith. 1.575712 of 37,645 *Viz.* 37 *l.* 12 *s.* 10¾ *d.* is the Diſcompt required, As above; which being taken from the 956 *l.* 10 *s.* leaves 918 *l.* 17 *s.* 1¼ *d.* the Sum which is to be paid in ready Money; as may be tried, by making it a Principal, and then finding what it would Amount to in ¾ of a Year at the Rate of Five and a Half *per Cent.* the which, by *Caſe* 1. *Page* 53. will be found to be juſt 956 *l.* 10 *s.* Therefore, &c.

If theſe Two Rules, and their Examples, be a little conſider'd, I preſume it will be very eaſie to conceive how to find the true Diſcompt or Rebate of any One propoſed Sum of Money, Due at the End of any given Interval of Time, and any given Rate, either of Simple, or Compound Intereſt, according as the Queſtion is propoſed.

And if it be required to find the whole Diſcompt of ſeveral Sums, Due at the End of ſeveral different Intervals of Time, it is but Computing them at ſo many ſeveral Operations, and then the Sum of all thoſe particular Diſcompts being taken from the Total Sum of all the Debts, will leave the true Diſcompt requir'd.

As for Inſtance;

Suppoſe A *were indebted to* B 750 l. *to be paid at Three ſeveral Payments, in this manner*; viz. 250 l. *at the End of One Year and a Half,* 100 l. *to be paid at the End of Two Years, and* 400 l. *at the End of Four Years; The Queſtion is, To find how much of the* 750 l. B, *ought to Rebate or Diſcompt, at Six* per Cent. per Annum *Simple Intereſt, to have his Whole Debt diſcharged by* A *in preſent Money?*

According to the Data's in this Queſtion, the particular Diſcompts (found by the Firſt Rule in this Section) will be theſe following:

The Diſcompt of 250 *l.* for 1,5 Year is 20,6423
The Diſcompt of 100 *l.* for 2 Years is 10,7142 Add
The Diſcompt of 400 l. for 4 Years is 77,4193

Conſequently the whole Diſcompt is 108,7758 Then if from 750 *l.* the whole Debt, there be Subſtracted 108,7758 the whole Diſcompt, there will Remain 641,2242 = 641 *l.* 4 *s.* 5¼ *d.* which is the true Sum that *A*, muſt give to *B*, in preſent Money, to be Diſcharg'd of his Debt.

Again, Suppoſe the ſame things given as above, and let it be requir'd to find the particular Diſcompts, &c. at Compound Intereſt.

Then, by the Second Rule in this Section, the Work will ſtand thus:

The Diſcompts of 250 *l.* for 1,5 Year is 20,9238
The Diſcompts of 100 *l.* for 2 Year is 11,0003 Add
The Diſcompts of 400 *l.* for 4 Year is 83,1633
Sum is 115,0874

Then 115,0874 Taken from 750 *l.* Leaves 634,9126 *Viz.* 634 *l.* 18 *s.* 3 *d.* is now the Sum which *A* is to give *B*, in preſent Money, to be clear of his Debt.

This is ſo plain to be underſtood, that I preſume it's needleſs to ſay any more in Proof of it, than what has been already ſaid in Page 115, and 116.

And from hence Naturally flows the following Method of finding the true Equated Time, wherein ſeveral Sums, Due at ſeveral Intervals of Time, may be paid at one entire Payment, without any Loſs, either to the Creditor or Debtor.

Sect. 2. The Equated Time of Payments truly Determin'd.

The uſual Rule laid down in divers Treatiſes of Arithmetick and Intereſt, *&c.* for finding of an Equated Time, for the Payment of ſeveral Sums of Money Due at the End of unequal Intervals of Time, is to this Effect.

Rule. { *Multiply every ſingle Sum of Money with the Time it becomes Due; and Divide the Sum of thoſe Products by the Total Debt; and the Quotient will be the true Time* (ſay they) *at which all the Money ought to be paid.*

I ſhall paſs over all the Arguments that are made uſe of *pro* and *con*, by Mr. *John Kerſey*, in his Book of Arithmetick, and by Sir *Samuel Morland*, in his Doctrine of Intereſt, and other Authors, about the Erroneouſneſs of this Rule; As alſo the Rules they lay down inſtead of it; and ſhall only proceed to ſhew how the true Eqnated Time may be found, from what hath been already done and proved.

When any Number of Payments are propoſed to be paid off at one entire Sum; Then, in order to determine the Equated Time for the Payment of that Sum, you muſt firſt find the particular Diſcompts of all the propoſed Payments, whoſe Times are to be Equated, according to the given Intervals of thoſe Paymenrs, at any Rate, either of Simple, or Compound Intereſt, as ſhall be agreed on by the Parties concern'd; and by thoſe Diſcompts find the preſent Sum that would Clear the Debt, if it were immediately paid; (*As above.*)

Then { *If the Sum of all thoſe particular Diſcompts, be Divided by the Product of the preſent Sum Multiplied with the Ratio of the ſame Rate of Intereſt by which thoſe Diſcompts were Computed, the Quotient will ſhew the true Equated Time required at Simple Intereſt.* }

For an Example of this Rule ;

Let it be requir'd to find the true Equated Time, wherein the aforeſaid three Sums, Due from A, *to* B (As in the laſt Inſtance) *may be ſafely paid without Loſs to either,* &c.

There the Sum of all the particular Diſcompts at Six *per Cent.* Simple Intereſt, is found to be 108,7758 and 750 — 108,7758 = 641,2242 the preſent Sum, if it were to be immediately paid.

Then, by the Rule above, 641,2242 × 0,06 = 38,47345 will be the Diviſor ; And 108,7758 the Dividend. Then 38,47345) 108,7758 (2,8273 is the Quotient ; That is, 2 Years and 302 Days will be the Time when *A*, may pay unto *B* his whole Debt of 750 *l.* without any Loſs to either of them.

The ſame found by *Logarithms.*

Sum of *Diſcompts* 108.7758 its Log. 2.036544

Preſent Worth is 641,2242 its Log. 2.807010 } Add
Ratio of the *Rate* is 0,06 its Log. $\bar{8}$.778151 }

From the 1ſt Log. Subſtr. this Log. 1.585161 Sum ;

There Remains the Logarithm 0.451383 of 2.8273 *Viz.* 2 Years and 302 Days will be the true Equated Time required by the Queſtion ; As before.

For if the Diſcompts be truly found, then it cannot be denied, but that the Preſent Worth of the whole Debt may by them be truly found ; And, I ſay, if that Preſent Worth be made a Principal, and the Equated Time (*as here found*) be made the Time of that Principal's being forborn at the ſame Rate of Intereſt, it will be found (*per Caſe* 1. *Pag.* 23) to Amount to juſt the whole Debt ; Therefore, the Equated Time is here truly found.

And if it be required to find the Equated Time at any propoſed Rate of Compound Intereſt, you muſt proceed in the ſame manner to find all the particular Diſcompts of the given Sums, and by them the preſent Sum that would Clear the Debt, if it were immediately paid; As in Pag. 115.

Then { *If the Log. of the preſent Sum be taken from the Log. of the Total Sum of all the Debts, and the Remainder be Divided by the Log. of R, the Amount of* 1 l. *at the ſame Rate of Intereſt by which thoſe Diſcompts were Computed; The Quotient will ſhew the Equated Time requir'd.*

As for Example.

Suppoſe it were requir'd to find the Equated Time at Compound Intereſt, wherein the 750 l. *Due from* A *to* B, *in the ſame manner as before, may be paid at one entire Payment, without Loſs,* &c.

Then the Sum of all the particular Diſcompts, found at 6 *per Cent*, Compound Intereſt, is 115,0874
And 750 — 115,0874 = 634,9126 is the preſent Sum that would Diſcharge the Debt, if it were immediately paid, as in Page 117. Then, by the Rule above, it
will be 750 its Logarithm is 2.875061 } Subſt.
Preſent Sum 634,9126 its Log. is 2.802714 }
R = 1,36 its Log. is 0.025306) 0.072347 (2,8588
That is, 2 Years, and 313 Days, is here found to be the Equated Time that *A* muſt Pay the 750 *l.* unto *B*, at one entire Payment; which is but 11 Days more than the Time found at Simple Intereſt.

And the Truth of the Equated Time thus found at Compound Intereſt, may be eaſily proved by the Help of *Caſe* 1. *Page* 53, *&c.* in the ſame manner as that of Simple Intereſt was in the laſt Page.

FINIS.

A Table of **Logarithms**, for *Numbers* Increaſing in their proper Order, from 1, To 10000. With their *Differences*.

Num	*Logarith.*	*Num*	*Logarith.*	*Num*	*Logarith.*
1	0.000000	34	1.531479	67	1.826075
2	0.301030	35	1.544068	68	1.832509
3	0.477121	36	1.556302	69	1.838849
4	0.602060	37	1.568202	70	1.845098
5	0.698970	38	1.579784	71	1.851258
6	0.778151	39	1.591065	72	1.857332
7	0.845098	40	1.602060	73	1.863323
8	0.903090	41	1.612784	74	1.869232
9	0.954242	42	1.623249	75	1.875061
10	1.000000	43	1.633468	76	1.880814
11	1.041393	44	1.643453	77	1.886491
12	1.079181	45	1.653212	78	1.892094
13	1.113943	46	1.662758	79	1.897627
14	1 146128	47	1.672098	80	1.903090
15	1 176091	48	1.681241	81	1.908485
16	1.204120	49	1.690196	82	1.913814
17	1.230449	50	1.698970	83	1.919078
18	1.255272	51	1.707570	84	1.924279
19	1.278754	52	1.716003	85	1.929419
20	1.301030	53	1.724276	86	1.934498
21	1.422219	54	1.732394	87	1.939519
22	1.342423	55	1.740363	88	1.944483
23	1.361728	56	1.748188	89	1.949390
24	1.380211	57	1.755875	90	1.954242
25	1.397940	58	1.763428	91	1.959041
26	1.414973	59	1.770852	92	1.963788
27	1.431364	60	1.778151	93	1.968483
28	1.447158	61	1.785330	94	1.973128
29	1.462398	62	1.792392	95	1.977724
30	1.477121	63	1.799341	96	1.982271
31	1.491362	64	1.806180	97	1.9867 2
32	1.505150	65	1.812913	98	1.991226
33	1.518514	66	1.819544	99	1.995635

A Table of Logarithms,

Num	0	1	2	3	4	*Diff*
100	000000	000434	000868	001301	001734	432
101	004321	004751	005181	005609	006038	428
102	008600	009026	009451	009876	010300	424
103	012837	013259	013679	014100	014520	420
104	017033	017451	017868	018284	018700	416
105	021189	021603	022016	022428	022841	411
106	025306	025715	026124	026533	026942	408
107	029384	029789	030195	030600	031004	404
108	033424	033826	034227	034628	035029	401
109	037426	037825	038223	038620	039017	397
110	041393	041787	042182	042575	042969	393
111	045323	045714	046105	046495	046885	390
112	049218	049606	049993	050380	050766	386
113	053078	053463	053846	054230	054613	383
114	056905	057286	057666	058046	058426	379
115	060698	061075	061452	061829	062206	376
116	064458	064832	065206	065580	065953	373
117	068186	068557	068928	069298	069668	370
118	071882	072250	072617	072985	073352	366
119	075547	075912	076276	076640	077004	364
120	079181	079543	079904	080266	080626	361
121	082785	083144	083503	083861	084219	357
122	086360	086716	087071	087426	087781	355
123	089905	090258	090611	090963	091315	352
124	093422	093772	094122	094471	094820	349
125	096910	097257	097604	097951	098297	347
126	100371	100715	101059	101403	101747	344
127	103804	104145	104487	104828	105169	341
128	107210	107549	107888	108227	108565	338
129	110590	110926	111262	111598	111934	336
130	113943	114277	114611	114944	115278	332
131	117271	117603	117934	118265	118595	330
132	120574	120903	121231	121560	121888	328
133	123852	124178	124504	124830	125156	325
134	127105	127429	127752	128076	128399	323
135	130334	130655	130977	131298	131618	321

Num	5	6	7	8	9	Di
100	002166	002598	003029	003460	003891	43
101	006466	006894	007321	007748	008174	42
102	010724	011147	011570	011993	012415	42
103	014940	015360	015779	016197	016615	42
104	019116	019532	019947	020361	020775	41
105	023252	023664	024075	024486	024896	41
106	027350	027757	028164	028571	028978	40
107	031408	031812	032216	032619	033021	40
108	035430	035830	036229	036629	037028	40
109	039414	039810	040207	040602	040998	39
110	043362	043755	044148	044540	044931	39
111	047275	047664	048053	048442	048830	39
112	051152	051538	051924	052309	052694	38
113	054996	055378	055760	056142	056524	38
114	058805	059185	059563	059942	060320	37
115	062582	062958	063333	063708	064083	37
116	066326	066698	067071	067443	067814	37
117	070038	070407	070776	071145	071514	37
118	073718	074085	074451	074816	075182	36
119	077368	077731	078094	078457	078819	36
120	080987	081347	081707	082067	082426	36
121	084576	084934	085291	085647	086004	35
122	088136	088490	088845	089198	089552	35
123	091667	092018	092370	092721	093071	35
124	095169	095518	095866	096215	096562	34
125	098644	098990	099335	099681	100026	34
126	102091	102434	102777	103119	103462	34
127	105510	105851	106191	106531	106870	34
128	108903	109241	109578	109916	110253	33
129	112270	112605	112940	113275	113609	33
130	115610	115943	116276	116608	116940	33
131	118926	119256	119586	119915	120245	33
132	122216	122543	122871	123198	123525	32
133	125481	125806	126131	126456	126781	32
134	128722	129045	129368	129690	130012	32
135	131939	132260	132580	132900	133219	32

A Table of Logarithms,

Num	0	1	2	3	4	Diff
136	133539	133858	134177	134496	134814	319
137	136721	137037	137354	137671	137987	316
138	139879	140194	140508	140822	141136	314
139	143015	143327	143639	143951	144263	311
140	146128	146438	146748	147058	147367	309
141	149219	149527	149835	150142	150449	307
142	152288	152594	152900	153205	153510	305
143	155336	155640	155943	156246	156549	303
144	158362	158664	158965	159266	159867	301
145	161368	161667	161967	162266	162564	298
146	164353	164650	164947	165244	165541	297
147	167317	167613	167908	168203	168497	295
148	170262	170555	170848	171141	171434	292
149	173186	173478	173769	174060	174351	290
150	176091	176381	176670	176959	177248	289
151	178977	179264	179552	179839	180126	287
152	181844	182129	182415	182700	182985	285
153	184691	184975	185259	185542	185825	283
154	187521	187803	188084	188366	188647	281
155	190332	190612	190892	191171	191451	279
156	193125	193403	193681	193959	194237	278
157	195900	196176	196453	196729	197005	276
158	198657	198932	199206	199481	199755	274
159	201397	201670	201943	202216	202488	272
160	204120	204391	204663	204934	205204	271
161	206826	207096	207365	207634	207904	269
162	209515	209783	210051	210319	210586	267
163	212188	212454	212720	212986	213253	266
164	214844	215109	215373	215638	215902	264
165	217484	217747	218010	218273	218536	262
166	220108	220370	220631	220892	221153	261
167	222716	222976	223236	223496	223755	260
168	225309	225568	225826	226084	226342	258
169	227887	228144	228400	228657	228913	257
170	230449	230704	230960	231215	231470	254
171	232996	233250	233504	233757	234011	253

Num	5	, 6	7	8	9	*Diff*
136	135133	135451	135768	136086	136403	31
137	138303	138618	138934	139249	139564	31
138	141450	141763	142076	142389	142702	31
139	144574	144885	145196	145507	145818	31
140	147676	147985	148294	148603	148911	30
141	150756	151063	151370	151676	151982	30
142	153815	154119	154424	154728	155032	30
143	156852	157154	157457	157759	158061	30
144	159868	160168	160469	160769	161068	30
145	162863	163161	163459	163758	164055	29
146	165838	166134	166430	166726	167022	97
147	168792	169086	169380	169764	169968	29
148	171726	172019	172311	172603	172895	29
149	174641	174932	175222	175512	175802	29
150	177536	177825	178113	178401	178689	28
151	180413	180699	180986	181272	181558	28
152	183270	183555	183839	184123	184407	28
153	186108	186391	186674	186956	187239	28
154	188928	189210	189490	189771	190051	28
155	191730	192010	192289	192567	192846	27
156	194514	194792	195069	195346	195623	27
157	197281	197556	197832	198107	198382	27
158	200029	200303	200577	200850	201124	27
159	202761	203033	203305	203577	203848	27
160	205475	205746	206016	206287	206556	27
161	208173	208441	208710	208978	209247	26
162	210853	211120	211388	211654	211921	26
163	213518	213783	214049	214314	214579	26
164	216166	216430	216694	216957	217221	26
165	218798	219060	219323	219585	219846	26
166	221414	221675	221936	222196	222456	26
167	224015	224274	224533	224792	225051	26
168	226600	226858	227115	227372	227630	25
169	229170	229426	229682	229938	230193	25
170	231724	231979	232233	232488	232742	25
171	234264	234517	234770	235023	235276	25

A Table of Logarithms,

Num	0	1	2	3	4	Diff
172	235528	235781	236033	236285	236537	252
173	238046	238297	238548	238799	239049	250
174	240549	240799	241048	241297	241546	249
175	243038	243286	243534	243782	244030	247
176	245513	245759	246006	246252	246499	246
177	247973	248219	248464	48709	248954	244
178	250420	250664	250908	251151	251395	243
179	252853	253096	253338	253580	253822	242
180	255272	255514	255755	255996	256236	241
181	257679	257918	258158	258398	258637	239
182	260071	260310	260548	260787	261025	238
183	262451	262688	262925	263162	263399	237
184	264818	265054	265290	265525	265761	235
185	264172	267406	267641	267875	268110	234
186	269513	269746	269980	270213	270446	233
187	271842	272074	272306	272538	272770	231
188	274158	274389	274620	274850	275081	230
189	276462	276691	276921	277151	277380	229
190	278754	278982	279210	279439	279667	228
191	281033	281261	281488	281715	281942	227
192	283301	283527	283753	283979	284205	226
193	285557	285782	286007	286232	286456	225
194	287802	288025	288249	288473	288696	224
195	290035	290257	290480	290702	290925	222
196	292256	292478	292699	292920	293141	221
197	294466	294687	294907	295127	295347	220
198	296665	296884	297104	297323	297542	219
199	298853	299071	299289	299507	299725	218
200	301030	301247	301464	301681	301898	217
201	303196	303412	303628	303844	304059	216
202	305351	305566	305781	305996	306210	215
203	307496	307710	307924	308137	308351	213
204	309630	309843	310056	310268	310481	212
205	311754	311966	312177	312389	312600	211
206	313867	314078	314289	314499	314710	210
207	315970	316180	316390	316599	316809	209

Num.	5	6	7	8	9	Dif
172	236789	237041	237292	237544	237795	252
173	239299	239550	239800	240050	240300	250
174	241795	242044	242293	242541	242790	249
175	244277	244524	244772	245019	245266	247
176	246745	246991	247236	247482	247728	246
177	249198	249443	249687	249932	250176	244
178	251638	251881	252125	252367	252610	243
179	254064	254306	254548	254790	255031	242
180	256477	256718	256958	257198	257439	241
181	258877	259116	259355	259594	259833	239
182	261263	261501	261738	261976	262214	238
183	263636	263873	264109	264345	264582	237
184	265996	266232	266467	266702	266937	235
185	268344	268578	268812	269046	269279	234
186	270679	270912	271144	271377	271609	233
187	273001	273233	273464	273696	273927	231
188	275311	275542	275772	276002	276232	230
189	277609	277838	278067	278296	278525	229
190	279895	280123	280351	280578	280806	228
193	282169	282395	282622	282849	283075	227
192	284431	284656	284882	285107	285332	226
193	286681	286905	287130	287354	287578	225
194	288920	289143	289366	289589	289812	224
195	291147	291369	291591	291813	292034	222
196	293363	293583	293084	294025	294245	221
197	295567	295787	296007	296225	296446	220
198	297760	297979	298198	298416	298635	219
199	299942	300160	300378	300595	300813	218
200	302114	302331	302547	302764	302980	217
201	304275	304490	304706	304921	305136	216
202	306425	306639	306854	307068	307282	215
203	308564	308778	308991	309204	309417	213
204	310693	310906	311118	311330	311542	212
205	312812	313023	313234	313445	313656	211
206	314920	315130	315340	315551	315760	210
207	317018	317227	317436	317646	317854	209

A Table of Logarithms,

Num	0	1	2	3	4	Diff
208	318063	318272	318481	318689	318898	209
209	320146	320354	320562	320769	320977	207
210	322219	322426	322633	322839	323046	106
211	324282	324488	324694	324899	325105	205
212	326336	326541	326745	326950	327155	204
213	328380	328583	328787	328991	329194	203
214	330414	330617	330819	331022	331225	202
215	332438	332640	332842	333044	333246	202
216	334454	334655	334856	335057	235257	201
217	336460	336660	336860	337060	337260	200
218	338456	338656	338855	339054	339253	199
219	340444	340642	340841	341039	341237	198
220	342423	342620	342817	343014	343212	197
221	344392	344589	344785	344981	345178	196
222	346353	346549	346744	346939	347135	195
223	348305	348500	358694	348889	349083	194
224	350248	350442	350636	350829	351023	193
225	352183	352375	352568	352761	352954	193
226	354108	354301	354493	354685	354876	192
227	356026	356217	356408	356599	356790	191
228	357935	358125	358316	358506	358696	190
229	359835	360025	360215	360404	360593	189
230	361728	351917	362105	362294	362482	188
231	363612	363800	363988	364176	364363	188
232	365488	365675	365862	366049	366236	187
233	367356	367542	367729	367915	368101	186
234	369216	369401	369587	369772	369958	185
235	371068	371253	371437	371622	371806	184
236	372912	373096	373280	373464	373647	184
237	374748	374932	375115	375298	375481	183
238	376577	376759	376942	377124	377306	182
239	378398	378580	378761	378953	379124	181
240	380211	380392	380573	380754	380934	181
241	382017	382197	382377	382557	382737	180
242	383815	383995	384174	384353	384533	179
243	385606	385785	385964	386142	386321	178

Num	5	6	7	8	9	Diff
208	319106	319314	319522	319730	319938	209
209	321184	321391	321598	321805	322012	207
210	323252	323458	323665	323871	324077	206
211	325310	325516	325721	325926	326131	205
212	327359	327563	327767	327972	328176	204
213	329398	329601	329805	330008	330211	203
214	331427	331630	331832	332034	332236	202
215	333447	333649	333850	335051	334253	202
216	335458	335658	335859	336059	336260	201
217	337459	337659	337858	338058	348257	200
218	339451	339650	339849	340047	340246	199
219	341435	341632	341830	342028	342225	198
220	343409	343606	343802	343999	344196	197
221	345374	345570	345766	345962	346157	196
222	347330	347525	347720	347915	348110	195
223	349278	349572	349666	349860	350054	194
224	351216	351410	351603	351796	351989	193
225	353147	353339	353532	353724	353916	193
226	355068	355260	355452	355643	355834	192
227	356981	357172	357363	357554	357744	191
228	358886	359076	359266	359456	369646	19
229	360783	360972	361161	361351	361539	189
230	362571	362859	363048	363236	363424	188
231	364551	364739	364926	365113	365301	188
232	366423	366610	366796	366983	367169	187
233	368287	368473	368659	368845	369030	186
234	370143	370328	370513	370698	370883	18
235	371991	372175	372360	372544	372728	184
236	373831	374015	374198	374382	374565	184
237	375664	375846	376029	376212	376394	18
238	377488	377670	377852	378034	378216	18
239	379306	379487	379668	379849	380030	18
240	381115	381296	381476	381656	381837	18
241	382917	383097	383277	383456	383636	180
242	384712	384891	385070	385249	385427	17
243	386499	386677	386856	387034	387212	17

A Table of Logarithms,

Num	0	1	2	3	4	Diff
244	387390	387568	387746	387923	388101	178
245	389166	389343	389521	389697	389875	177
246	390935	391112	391288	391464	391641	176
247	392697	392873	393048	393224	393400	176
248	394452	394627	394802	394977	395152	175
249	396199	396374	396548	396722	396896	174
250	397940	398114	398287	398461	398634	173
251	399674	399847	400020	400192	400365	173
252	401400	401573	401745	401917	402089	172
253	403120	403292	403464	403635	403807	171
254	404834	405005	405176	405346	405517	171
255	406540	406710	406881	407051	407221	170
256	408240	408410	408579	408749	408918	169
257	409933	410102	410271	410440	410608	169
258	411620	411788	411956	412124	412292	168
259	413300	413467	413635	413802	413970	167
260	414973	415140	415307	415474	515641	167
261	416640	416807	416973	417139	417306	166
262	418301	418467	418633	418798	418964	165
263	419956	420121	420286	420451	420616	165
264	421604	421768	421933	422097	422261	164
265	423246	423410	423573	423737	423901	164
266	424882	425045	425208	425371	425534	163
267	426511	426674	426836	426999	427161	162
268	428135	428297	428459	428621	428782	162
269	429752	429914	430075	430236	430398	161
270	431364	431525	431685	431846	432007	160
271	432969	433129	433290	433450	433610	160
272	434569	434728	434888	435048	435207	159
273	436163	436322	436481	436640	436798	159
274	437751	437909	438067	438226	438384	158
275	439333	439491	439648	439806	439964	158
276	440909	441066	441224	441381	441538	157
277	442480	442637	442793	442950	443106	157
278	444045	444201	444357	444513	444669	156
279	445604	445760	445915	446071	446226	155

Num	5	6	7	8	9	Dif
244	388279	388456	388634	388811	388989	178
245	390051	390228	390405	390582	390758	177
246	391817	391993	392169	392345	392521	176
247	393575	393751	393926	394101	394176	176
248	395326	395501	395676	395850	396025	17
249	397070	397245	397418	397592	397766	174
250	398808	398981	399154	399327	399501	173
251	400538	400711	400883	401056	401228	173
252	402261	402433	402605	402777	402949	172
253	403978	404149	404320	404492	404663	171
254	405688	405858	406029	406199	406370	171
255	407391	407561	407731	407900	408070	170
256	409087	409257	409426	409595	409764	169
257	410777	410946	411114	411283	411451	169
258	412460	412628	512796	412964	413132	168
259	014137	414305	414472	414639	414806	167
260	415808	415974	416141	416308	416474	167
261	417472	417638	417804	417970	418135	166
262	419129	419295	419460	419625	419791	165
263	420781	420945	421110	421275	421439	165
264	422426	422590	422754	422918	423082	164
265	424064	424228	424392	424555	424718	164
266	425697	425860	426023	426186	426349	163
267	427324	427486	427648	427811	427973	162
268	428944	429106	429268	429429	429591	162
269	430559	430720	430881	431042	431203	161
270	432167	432328	432488	432649	432809	160
271	433770	433930	434090	434249	434409	160
272	435366	435526	435685	435844	436003	159
273	436957	437116	437275	437433	437592	159
274	438542	438700	438859	439017	439175	158
275	440122	440279	440437	440594	440752	158
276	441695	441852	442009	442166	442323	157
277	443263	443419	443576	443732	443889	157
278	444825	444981	445137	445293	445449	156
279	446382	446537	446692	446848	447003	155

Num	0	1	2	3	4	*Diff*
280	447158	447313	447468	447623	447778	155
281	448706	448861	449015	449170	449324	154
282	450249	450403	450557	450711	450865	154
283	451785	451940	452093	452247	452400	153
284	453318	453471	453624	453777	453930	153
285	454845	454997	455149	455302	455454	152
286	456366	456518	456670	456821	456973	152
287	457882	458033	458184	458336	458487	151
288	459392	459543	459694	459845	459995	151
289	460898	461048	461198	461348	461499	150
290	462398	452548	462697	462847	462997	150
291	463893	464042	464191	464340	464489	149
292	465383	465532	465680	465829	465977	149
293	466868	467016	457164	467312	457460	148
294	468347	468495	468643	468790	468938	147
295	469822	469969	470116	470263	470410	147
296	471292	471438	471585	471732	471878	146
297	472756	472903	473049	473195	473341	146
298	474216	474362	474508	474653	474799	146
299	475671	475816	475962	476107	476252	145
300	477121	477266	477411	477555	477700	145
301	478566	478711	478855	478999	479143	144
302	480007	480151	480294	480438	480582	144
303	481443	481586	481729	481872	482016	143
304	482874	483016	483159	483302	483445	143
305	484300	484442	484585	484727	474869	142
306	485721	485863	486005	486147	486289	142
307	487138	487280	487421	487563	487704	141
308	488551	488692	488833	488874	489114	141
309	489958	490099	490239	490380	490520	140
310	491362	491502	491642	491782	491922	140
311	492760	492900	493040	493179	493319	139
312	494155	494294	494433	494572	494711	139
313	495544	495683	495822	495960	496099	139
314	496930	497068	497206	497344	497482	138
315	498311	498448	498586	498724	498862	138

Num	5	6	7	8	9	Diff
280	447933	448088	448242	448397	448552	155
281	449478	449633	449787	449941	450095	154
282	451018	451172	451326	451479	451633	154
283	452553	452706	452859	453012	453165	153
284	454082	454235	454387	454540	454692	153
285	455606	455758	455910	456062	456214	152
286	457125	457276	457428	457579	457731	152
287	458638	458789	458940	459091	459242	151
288	460146	460296	460447	460597	460748	151
289	461649	461799	461948	462098	462248	150
290	463146	463296	463445	463594	463744	150
291	464639	464787	464936	465085	465234	149
292	466126	466274	466423	466571	466719	149
293	467608	467756	467904	468052	468200	148
294	469085	469233	469380	469527	469675	147
295	470557	470704	470851	470998	471145	147
296	472025	472171	472318	472464	472610	146
297	473487	473633	473779	473925	474071	146
298	474944	475090	475235	475381	475526	14
299	476397	476542	476687	476832	476976	145
300	477844	477989	478133	478278	478422	145
301	479287	479431	479575	479719	479863	144
302	480725	480869	481012	481156	481299	144
303	482159	482302	482445	482588	482731	143
304	483587	483730	483872	484015	484157	143
305	485011	485153	485295	485437	485579	142
306	486430	486572	486714	486855	486997	142
307	487845	487986	488127	488269	488410	141
308	489255	489396	489537	489677	489818	141
309	490661	490801	490941	491081	491222	140
310	492062	492201	492341	492481	492621	140
311	493458	493597	493737	493876	494015	139
312	494850	494989	495128	495267	495406	139
313	496237	496376	496514	496653	496791	139
314	497621	497759	497897	498035	498173	138
315	498999	499137	499275	499412	499550	138

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Num	0	1	2	3	4	Diff
316	499687	499824	499962	500099	500236	137
317	501059	501196	501333	501470	501607	137
318	502427	502564	502700	502837	502973	136
319	503791	503927	504063	504199	504335	136
320	505150	505286	505421	505557	505692	136
321	506505	506640	506775	506911	507046	135
322	507856	507991	508125	508260	508395	135
323	509202	509337	509471	508606	509740	134
324	510545	510679	510813	510947	511081	134
325	511883	512017	512150	512284	512417	134
326	513218	513351	513484	513617	513750	133
327	514548	514681	514813	514946	515079	132
328	515874	516006	516139	516271	516403	132
329	517196	517328	517460	517592	517724	131
330	518514	518645	518777	518909	519040	131
331	519828	519959	520090	520221	520352	131
332	521138	521269	521400	521530	521661	131
333	522444	522575	522705	522835	522966	130
334	523746	523875	524006	524136	524256	130
335	525045	525174	525304	525433	525563	129
336	526339	526468	526598	526727	526856	129
337	527630	527759	527888	528016	528145	129
338	528917	529045	529174	529302	529430	128
339	530200	530328	530456	530584	530712	128
340	531479	531607	531734	531862	531989	128
341	532754	532882	533009	533136	533263	128
342	534026	534153	534280	534407	534534	127
343	535294	535421	535547	535674	535800	127
344	536558	536685	536811	536937	537063	127
345	537819	537945	538071	538197	538322	126
346	39076	539202	539327	539452	539578	125
347	540329	540455	540580	540705	540830	125
348	541579	541704	541829	541953	542078	125
349	542825	542950	543074	543199	543323	124
350	544068	544192	544316	544440	544564	124
351	545307	545431	545555	545678	545802	124

Num	5	6	7	8	9	Di
316	500374	500511	500648	500785	500922	13
317	501744	501880	502017	502154	502290	13
318	503109	503246	503382	503518	503654	13
319	504471	504607	504743	504878	505014	13
320	505828	505963	506099	506234	506370	13
321	507181	507316	507451	507586	507721	33
322	508530	508664	508799	508933	509068	13
323	509874	510008	510143	510277	510411	13
324	511215	511348	511482	511616	511750	13
325	512551	512684	512818	512951	513084	13
326	513883	514016	514149	514282	514415	13
327	515211	515344	515476	515609	515741	13
328	516535	516668	516800	516932	517064	13
329	517855	517987	518119	518251	518382	13
330	519171	519303	519434	519566	519697	13
331	520483	520614	520745	520876	521007	13
332	521792	521922	522053	522183	522314	13
333	523096	523226	523356	523486	523616	13
334	524396	524526	524656	524785	524915	13
335	525692	525822	525951	526081	526210	12
336	526984	527114	527243	527372	527501	12
337	528274	528402	528531	528660	528788	12
338	529559	529687	529815	529943	530072	12
339	530840	530968	531095	531223	531351	12
340	532117	532245	532372	532500	532627	12
341	533391	533518	533645	533772	533899	12
342	534661	534787	534914	535041	535167	12
343	535927	536053	536179	536306	536432	12
344	537189	537315	537441	537567	537693	12
345	538448	538574	538699	538825	538951	12
346	539703	539829	539954	540079	540204	12
347	540955	541080	541205	541330	541454	12
348	542203	542327	542452	542576	542701	12
349	543447	543571	543696	543820	543944	12
350	544688	544812	544936	545060	545183	12
351	545925	546049	546172	546296	546419	12

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Num	0	1	2	3	4	Diff
352	546543	546666	546789	546913	547036	123
353	547775	547898	548021	548144	548267	123
354	549003	549126	549249	549371	549494	123
355	550228	550351	550473	550595	550717	122
356	551450	551572	551694	551816	551938	122
357	552668	552790	552911	553033	553155	121
358	553883	554004	554126	554247	554368	121
359	555094	555215	555336	555457	555578	121
360	556303	556423	556544	556664	556785	120
361	557507	557628	557748	557868	557988	120
362	558709	558829	558948	559068	559188	120
363	559907	560026	560146	560265	560385	119
364	561101	561221	561340	561459	561578	119
365	562293	562412	562531	562650	562759	119
366	563481	563600	563718	563837	563955	119
367	564566	564784	564903	565021	565139	118
368	565848	565966	566084	566202	566320	118
369	567026	567144	567262	567379	567497	188
370	568202	568319	568436	568554	568671	117
371	569374	569491	569608	569725	569842	117
372	570543	570660	570776	570893	571010	117
373	571709	571825	571942	572058	572174	116
374	572872	572988	573104	573220	573336	116
375	574031	574147	574263	574379	574494	116
376	575188	575303	575419	575534	575650	115
377	576341	576457	576572	576687	576802	115
378	577492	577607	577722	577836	577951	115
379	578639	578754	578868	578983	579097	114
380	579784	579898	580012	580126	580240	114
381	580925	581039	581153	581267	581381	114
382	582063	582177	582291	582404	582518	113
383	583199	583312	583425	583539	583652	113
384	584331	584444	584557	584670	584783	113
385	585461	585573	585686	585499	585912	112
386	586587	586700	586812	586925	587037	112
387	587711	587823	587935	588047	588160	112

Num	5	6	7	8	9	Dif
352	547159	547282	547405	547529	547652	12
353	548389	548512	548635	548758	548881	12
354	549616	549739	549861	549984	550106	12
355	550840	550962	551084	551206	551328	12
356	552059	552181	552303	552425	552547	12
357	553276	553398	553519	553640	553762	121
358	554489	554610	554731	554852	554973	121
359	555699	555820	555940	556061	556182	121
360	556905	557026	557146	557267	557387	12
361	558108	558228	558349	558469	558589	12
362	559308	559428	559548	559667	559787	12
363	560504	560624	560743	560863	560982	11
364	561698	561817	561936	562055	562174	11
365	562887	563006	563125	563244	563362	11
366	564074	564192	564311	564429	564548	11
367	565257	565376	565494	565612	565730	11
368	566437	566555	566673	566791	566909	11
369	567614	567732	567849	567967	568084	11
370	568788	568905	569023	569140	569257	11
371	569959	570076	570193	570309	570426	11
372	571126	571243	571359	571476	571592	11
373	572291	572407	572523	572639	572755	11
374	573452	573568	573684	573800	573915	11
375	574610	574726	574841	574957	575072	11
376	575765	575880	575996	576111	576226	11
377	576917	577032	577147	577262	577377	11
378	578066	578181	578295	578410	578525	11
379	579212	579326	579441	579555	579669	11
380	580355	580469	580583	580697	580811	11
381	581494	581608	581722	581836	581950	11
382	582631	582745	582859	582972	583085	11
383	583765	583879	583992	584105	584218	11
384	584896	585009	585122	585235	585348	11
385	586024	586137	586250	586362	586475	11
386	587149	587262	587374	587486	587599	11
387	588272	588384	588496	588608	588720	11

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Num	0	1	2	3	4	Diff
388	588832	588944	589055	589167	589279	112
389	589950	590061	590173	590284	590396	111
390	591065	591176	591287	591398	591510	111
391	592177	592288	592399	592510	592621	111
392	593286	593397	593508	593618	593729	111
393	594393	594503	594613	594724	594834	111
394	595496	595606	595717	595827	595937	110
395	596597	596707	596817	596927	597037	110
396	597695	597805	597914	598024	598134	110
397	598790	598900	599009	599119	599228	109
398	599883	599992	600101	600216	600319	109
399	600973	601082	601190	601299	601408	109
400	602060	602168	602277	602386	602494	109
401	603144	603253	603361	603469	603577	108
402	604226	604334	604442	604550	604658	108
403	605305	605413	605521	605628	605736	107
404	606381	606489	606596	606704	606811	107
405	607455	607562	607669	607777	607884	107
406	608526	608633	608740	608847	608954	107
407	609594	609701	609808	609914	610021	107
408	610660	610767	610873	610979	611086	106
409	611723	611829	611936	612042	612148	106
410	612784	612890	612996	613101	613207	106
411	613842	613947	614053	614159	614264	106
412	614897	615003	615108	615213	615319	105
413	615950	616055	616160	616265	616370	105
414	617000	617105	617210	617315	617420	105
415	618048	618153	618257	618362	618466	105
416	619093	619198	619302	619406	619511	104
417	620136	620240	620344	620448	620552	104
418	621176	621280	621384	621488	621592	104
419	622214	622318	622421	622525	622628	104
420	623249	623353	623456	623559	623663	103
421	624282	624385	624488	624591	624695	103
422	625312	625415	625518	625621	625724	103
423	626340	66443	626546	626648	626751	103

Num	5	6	7	8	9	Dif
388	589391	589503	589615	589726	589838	112
389	590507	590619	590730	590842	590953	111
390	591621	591732	591843	591955	592066	111
391	592732	592843	592954	593064	593175	111
392	593840	593950	594061	594171	594282	111
393	594945	595055	595165	595276	595386	111
394	596047	596157	596267	596377	596487	110
395	597146	597256	597366	597476	597586	110
396	598243	598353	598462	598572	598681	110
397	599337	599446	599556	599665	599774	109
398	600428	600537	600646	600755	600864	109
399	601517	601625	601734	601843	601951	109
400	602603	602711	602819	602928	603036	109
401	603685	603794	603902	604010	604118	108
402	604766	604874	604982	605089	605197	108
403	605843	605951	606059	606166	606274	107
404	606918	607026	607133	607240	607348	107
405	607991	608098	608205	608312	608419	107
406	609060	609167	609274	609381	609488	107
407	610128	610234	610341	610447	610554	107
408	611192	611298	611405	611511	611617	106
409	612254	612360	612466	612572	612678	106
410	613313	613419	613525	613630	613736	106
411	614370	614475	614681	614586	614792	106
412	615424	615529	615634	615740	615845	105
413	616475	616580	616685	616790	616895	105
414	617524	617629	617734	617839	617943	105
415	618571	618676	618780	618884	618989	105
416	619615	619719	619823	619928	620032	104
417	620656	620760	620864	620968	621072	104
418	621695	621799	621903	622007	622110	104
419	622732	622835	622939	623042	623146	104
420	623766	623869	623973	624076	624179	103
421	624798	624901	625004	625107	625209	103
422	625827	625929	626032	626135	626238	103
423	626853	626956	627058	627161	627263	103

Num	0	1	2	3	4	Diff
424	627366	627468	627571	627673	627775	102
425	628389	628491	628593	628695	628797	102
426	629410	629511	629613	629715	629817	102
427	630428	630530	630631	630733	630835	102
428	631414	631545	631647	631748	631849	101
429	632457	632559	632660	632761	632862	101
480	633468	633569	633670	633771	633872	100
481	634477	634578	634679	634779	634880	100
482	635484	635584	635685	635785	635886	100
433	636488	636588	636688	636789	636889	100
434	637490	637590	637690	637790	637890	99
435	638489	638589	638689	638789	638888	99
436	639486	639586	639686	639785	639885	99
437	640481	640581	640680	640779	640879	99
438	641474	641573	641672	641771	641871	99
439	642465	642563	642662	642761	642860	99
440	643453	643551	643650	643749	643847	98
441	644439	644537	644636	644734	644832	98
442	645422	645521	645619	645717	645815	98
443	646404	646502	646600	646698	646796	98
444	647383	647481	647579	647676	647774	98
445	648360	648458	648555	648553	648750	97
446	649335	649432	649530	649627	649724	97
447	650308	650405	650502	650599	650696	97
448	651278	651375	651472	611569	651666	97
449	652246	652343	652440	652536	652633	97
450	653212	653309	653406	653502	653598	97
451	654176	654273	654369	654465	654562	96
452	655138	655234	655331	655427	655523	96
453	656098	656194	656290	656386	656481	96
454	657056	657151	657247	657343	657438	95
455	658011	658107	658202	658298	658393	95
456	658965	659060	659155	659250	659346	95
457	659916	660011	660106	660201	660296	95
458	660865	660960	661055	661150	661245	9
459	661813	661907	662002	662096	662191	94

Num	5	6	7	8	9	Diff
424	627878	627980	628082	628185	628287	102
425	628900	629002	629104	629206	629308	102
426	629919	630021	630123	630224	630326	102
427	630936	631038	631139	631241	631342	102
428	631951	632052	632153	632255	632356	101
429	632963	633054	633165	633266	633367	101
430	633973	634074	634175	634276	634376	100
431	634981	635081	635182	635283	635383	100
432	635986	636087	636187	636287	636388	100
433	636989	637089	637189	637289	637390	100
434	637990	638090	638190	638289	638389	99
435	638988	639088	639188	639287	639387	99
436	639984	640084	640183	640283	640382	99
437	640978	641077	641177	641276	641375	99
438	641970	642069	642168	642267	642366	99
439	642959	643058	643156	643255	643354	99
440	643946	644044	644143	644242	644340	98
441	644931	645029	645127	645226	645324	98
442	645913	646011	646109	646208	646306	98
443	646894	646992	647089	647187	647285	98
444	647872	647969	648067	648165	648262	98
445	648848	648945	649043	649140	649237	97
446	649821	649919	650016	650113	650210	97
447	650794	650890	650987	651084	651181	97
448	651760	651859	651956	652053	652150	97
449	652630	655826	652923	653019	653116	97
450	653695	653791	653888	653984	654080	97
451	654658	654754	654850	654946	655042	96
452	655619	655714	655810	655906	656002	96
453	656577	656673	656769	656864	656960	96
454	657534	657629	657725	657820	657916	96
455	658488	658584	658679	658774	658870	95
456	659441	659536	659631	659726	659821	95
457	660391	660486	660581	660676	660771	95
458	661339	661434	661529	661623	661718	94
459	662285	662380	662474	662569	662663	94

A Table of Logarithms,

Num	0	1	2	3	4	Dif
460	662758	662852	662947	663041	663135	94
461	663701	663795	663889	663983	664078	94
462	664642	664736	664830	664924	665018	94
463	665581	665675	665768	665862	665956	94
464	666518	666612	666705	666799	666892	94
465	667453	667546	667640	667733	667826	94
466	668385	668479	668572	668665	668758	94
467	669317	669410	669503	669596	669689	93
468	670246	670339	670431	670524	670617	93
469	671173	671265	671358	671451	671543	93
470	672098	672190	672283	672375	672467	93
471	673021	673113	673205	673297	673390	92
472	673942	674034	674126	674218	674310	92
473	674861	674953	675045	675136	675228	92
474	675778	675870	675961	676053	676145	91
475	676694	676785	676876	676968	677059	91
476	677607	677698	677789	677881	677972	91
477	678518	678609	678700	678791	678882	91
478	679428	679519	679610	679700	679791	91
479	680335	680426	680517	680607	680698	91
480	681241	681332	681422	681513	681603	90
481	682145	682235	682326	682416	682506	90
482	683047	683137	683227	683317	683407	90
483	683947	684037	684127	684217	684307	90
484	684845	684935	685025	685114	685204	90
485	685742	685831	685921	686010	686100	89
486	686636	686726	686815	686904	686994	89
487	687529	687618	687707	687796	687885	89
488	688420	688509	688598	688687	688776	89
489	689309	689398	689486	689575	689664	89
490	690196	690285	690373	690462	690550	89
491	691081	691170	691258	691347	691435	88
492	691965	692053	692142	692230	692318	88
493	692847	692935	693023	693111	693199	88
494	693727	693815	693903	693991	694078	88
495	694605	694693	694781	694868	694956	88

Num	5	6	7	8	9	Di
460	663230	663324	663418	663512	663607	94
461	664172	664266	664360	664454	664548	94
462	665112	665206	665399	665393	665487	94
463	666050	666143	666237	666331	666424	94
464	666986	667079	667173	667266	667359	94
465	667920	668013	668106	668199	668293	94
466	668852	668945	669038	669131	669224	94
467	669782	669874	669967	670060	670153	9
468	670710	670802	670895	670988	671080	9
469	671636	671728	671821	671913	672005	9
470	672560	672652	672744	672836	672929	9
471	673482	673574	673666	673758	673850	9
472	674402	674494	674586	674677	674769	9
473	675320	675412	675503	675595	675687	9
474	676236	676328	676419	676511	676602	91
475	677150	677242	687333	677424	677516	91
476	678063	678154	678245	678336	678427	91
477	678973	679064	679155	679246	679337	91
478	679882	679973	680063	680154	680245	91
479	680789	680879	680970	681060	681151	91
480	681693	681784	681874	681964	682055	90
481	682596	682686	682777	682867	682957	90
482	683497	683587	683677	683767	683857	90
483	684396	684486	684576	684666	684756	90
484	685224	685383	685473	685563	685652	90
485	685169	686279	686358	686458	686547	89
486	687083	687172	687261	687351	687440	89
487	687975	688064	688153	688242	688331	89
488	688865	688953	649042	689131	689220	89
489	689753	689841	689930	690019	690107	89
490	690639	690727	690816	690905	690993	[illegible]
491	691524	691612	691700	691789	691877	88
492	692406	692494	692583	692671	692759	88
493	693287	693375	693463	693551	693639	88
494	694166	694254	694312	694430	694517	88
495	695044	695131	695219	695307	695394	88

A Table of Logarithms,

Num	0	1	2	3	4	Diff
496	695482	695569	695657	695744	695832	87
497	696356	696444	696531	696618	696706	87
498	697229	697317	697404	697491	697578	87
499	698101	698188	698275	698362	698449	87
500	698970	699057	699144	799 31	699317	87
501	699838	699924	700011	700098	700184	87
502	70 704	700790	700877	700963	701050	86
503	701568	701654	701741	701827	701913	86
504	702430	702517	702603	702689	702775	86
505	703291	703377	703463	703549	703635	86
506	704151	704236	704322	704408	704494	86
507	705008	705094	705179	705265	705350	86
508	705864	705949	706035	706120	706206	85
509	706718	706803	706888	706974	707059	85
510	707570	707655	707740	707826	707911	85
511	708421	708506	708591	708676	708761	85
512	709270	709355	709440	709524	709609	85
513	710117	710202	710287	710371	710456	85
514	710963	711048	711132	711217	711301	84
515	711807	711892	711975	712060	712144	84
516	712650	712734	712818	712902	712986	84
517	713491	713575	713659	713742	713826	84
518	714330	714414	714497	714581	714665	84
519	715167	715251	715335	715418	715502	84
520	716003	716087	716170	716254	716337	83
521	716838	716921	717004	717088	717171	83
522	717671	717754	717837	717920	718003	83
523	718502	718585	718668	718751	718834	83
524	719331	719414	719497	719580	719663	83
525	720159	720242	720325	720407	720490	83
526	720986	721068	721151	721233	721316	82
527	721811	721893	721975	722058	722140	82
528	722634	722716	722798	722881	722963	82
529	723456	723538	723620	723702	723784	82
530	724276	724358	724440	724522	724603	82
531	725094	725176	725258	725340	725422	82

Num	5	6	7	8	9	Di
496	695919	696007	696094	696182	696269	8
497	696793	696880	696968	697055	697142	8
498	697665	697752	697839	697926	698014	8
499	698535	698622	698709	698766	698883	8
500	699404	699491	699578	699664	699751	8
501	700271	700358	700444	700531	700617	8
502	701136	701222	701309	701395	701482	8
503	701999	702086	702172	702258	702344	8
504	702861	702947	703033	703119	703205	8
505	703721	703807	703893	703979	704065	8
506	704570	704665	704751	704837	704922	8
507	705436	705522	705607	705693	705778	8
508	706291	706376	706462	706547	706632	8
509	707144	707229	707315	707400	707485	8
510	707996	708081	708166	708251	708336	8
511	708846	708931	709015	709100	709185	8
512	709694	709779	709863	709948	710033	8
513	710540	710625	710710	710794	710879	8
514	711385	711470	711554	711639	711723	8
515	712229	712313	712397	712481	712566	8
516	713070	713154	713238	713323	713406	8
517	713910	713994	714078	714162	714246	8
518	714749	714833	714916	715000	715084	8
519	715586	715669	715753	715836	715920	8
520	716421	716504	716588	716671	716754	8
521	717254	717338	717421	717504	717587	8
522	718086	718169	718253	718336	718419	8
523	718917	719000	719083	716165	719248	8
524	719745	719828	719911	719994	720077	8
525	720573	720655	720738	720821	720903	8
526	721398	721481	721563	721646	721728	82
527	722222	722305	722387	722469	722552	82
528	723045	723127	723209	723291	723374	82
529	723866	723948	724030	724112	724194	82
530	724685	724767	724849	724931	725013	82
531	725503	725585	725667	725748	725830	82

A Table of **Logarithms**,

Num	0	1	2	3	4	Diff
532	725912	725993	726075	726156	726238	82
533	726727	726809	726890	726972	727053	81
534	727541	727623	727704	727785	727866	81
535	728354	728435	728516	728597	728678	81
536	729165	729246	729327	729408	729489	81
537	729974	730055	730136	730217	730298	81
538	730782	730863	730944	731024	731105	81
539	731589	731669	731750	731830	731911	81
540	732394	732474	732555	732635	732715	80
541	733197	733278	733358	733438	733518	80
542	733999	734079	734159	734240	734320	80
543	734800	734880	734960	735040	735120	80
544	735599	735679	735759	735838	735918	80
545	736397	736476	736556	736635	736715	80
546	737193	737272	737352	737431	737511	79
547	737987	738067	738146	738225	738305	79
548	738781	738860	738939	739018	739007	79
549	739572	739651	739731	739810	739889	79
550	740363	740442	740521	740599	740678	79
551	741152	741230	741309	741388	741467	79
552	741939	742018	742096	742175	742254	79
553	742725	742804	742882	742961	743039	78
554	743510	743588	743667	743745	743823	78
555	744293	744371	744449	744528	744606	78
556	745075	745153	745231	745309	745387	78
557	745855	745933	746011	746089	746167	78
558	746634	746712	746790	746868	746945	78
559	747412	747489	747567	747645	747722	78
560	748188	748266	748343	748421	748498	77
561	748963	749040	749118	749195	749272	77
562	749736	749814	749891	749968	750045	77
563	750508	750586	750663	750740	750817	77
564	751279	751356	751433	751510	751587	77
565	752048	752125	752202	752279	752356	77
566	752816	752893	752970	753047	753123	77
567	753583	753660	753736	753813	753889	77

Num	5	6	7	8	9	Di
532	726320	726401	726483	726564	726646	8
533	727134	727216	727297	727379	727460	8
534	727948	728029	728110	728191	728273	8
535	728759	728841	728922	729003	729084	8
536	729570	729651	729732	729813	729893	8
537	730378	730459	730540	730621	730702	8
538	731186	731266	731347	731428	731508	8
539	731991	732072	732152	732233	732313	8
540	732796	732876	732956	733037	733117	8
541	733598	733679	733759	733839	733919	8
542	734400	734480	734560	734640	734720	8
543	735199	735279	735359	735439	735519	8
544	735998	736078	736157	736237	736317	8
545	736795	736874	736954	737034	737113	8
546	737590	737670	737749	737829	737908	7
547	738384	738463	738543	738622	738701	7
548	739177	739256	739335	739414	739493	7
549	739968	740047	740126	740205	740284	7
550	740757	740836	740915	740994	741073	7
551	741546	741624	741703	741782	741860	7
552	742332	742411	742489	742568	742647	7
553	743118	743196	743275	743353	743431	7
554	743902	743980	744058	744136	744215	7
555	744684	744762	744840	744919	744997	7
556	745465	745543	745621	745699	745777	7
557	746245	746323	746401	746479	746556	7
558	747023	747101	747179	747256	747334	7
559	747800	747878	747955	748033	748110	7
560	748576	748653	748731	748808	748889	7
561	749350	749427	749504	749582	749659	7
562	750123	750200	750277	750354	750431	7
563	750894	750971	751048	751125	751202	7
564	751664	751741	751818	751895	751972	7
565	752433	752509	752586	752663	752740	7
566	753200	753277	753353	753430	753506	7
567	753966	754042	754119	754195	754272	7

A Table of Logarithms,

Num	0	1	2	3	4	Diff
568	754348	754425	754501	754578	754654	76
569	755112	755189	755265	755341	755417	76
570	755875	755951	756027	756103	756179	76
571	756636	756712	756788	756864	756940	76
572	757396	757472	757548	757624	757700	76
573	758155	758230	758306	758382	758458	76
574	758912	758988	759063	759139	759214	76
575	759668	759743	759819	759894	759970	75
576	760422	760498	760573	760649	760724	75
577	761176	761251	761326	761402	761477	75
578	761928	762003	762078	762153	762228	75
579	762679	762754	762829	762904	762978	75
580	763428	763503	763578	763653	763727	75
581	764176	764251	764326	764400	764475	75
582	764923	764998	765072	765147	765221	75
583	765669	765743	765818	765892	765966	74
584	766413	766487	766562	766635	766710	74
585	767156	767230	767304	767379	767453	74
586	767898	767972	768046	768120	768194	74
587	768638	768712	768786	768860	768934	74
588	769377	769451	769525	769599	769573	74
589	770115	770189	770263	770336	770410	74
590	770825	770926	770999	771073	771146	74
591	771587	771661	771734	771808	771881	73
592	772322	772395	772468	772542	772615	73
593	773055	773128	773201	773274	773348	73
594	773786	773860	773933	774006	775079	73
595	774517	774590	774663	774736	774809	73
596	775246	775319	775392	775465	775538	73
597	775974	776047	776120	776193	776265	73
598	776701	776774	776846	776919	776992	73
599	777427	777499	777572	777944	777717	72
600	778151	778224	778296	778368	778441	72
601	778874	778947	779019	779091	779163	72
602	779596	779669	779741	779813	779885	72
603	780317	780389	780461	780533	780905	72

Num	5	6	7	8	9
568	754730	754807	754883	754960	755036
569	756494	755570	755646	755722	755799
570	756256	756332	756408	756484	756560
571	757016	757092	757168	757244	757320
572	757775	757851	757927	758003	758079
573	758533	758609	758685	758761	758836
574	759290	759366	759441	759517	759592
575	760045	760121	760196	760272	760347
576	760799	750875	760950	761025	761101
577	761552	761627	761702	761778	761853
578	762303	762378	762453	762529	762604
579	763053	763128	763203	763278	763353
580	763802	763877	763952	764027	764101
581	764550	764624	764699	764774	764848
582	765296	765370	765445	765519	765594
583	766041	766115	766190	766264	766338
584	766784	766859	766933	767007	767082
585	767527	767601	767675	767749	767823
586	768268	768342	768416	768490	768564
587	769008	769082	769156	769230	769303
588	769746	769820	769894	769968	770042
589	770484	770557	770631	770705	770778
590	771220	771293	771367	771440	771514
591	771955	772028	772102	772175	772248
592	772688	772762	772835	772908	772981
593	773421	773494	773567	773640	773713
594	774152	774225	774298	774371	774444
595	774882	774955	775028	775100	775173
596	775610	775683	775756	775829	775902
597	776338	776411	776483	776556	776629
598	777064	777137	777209	777282	777354
599	777789	777862	777934	778006	778079
600	778513	778585	778658	778730	778802
601	779236	779308	779380	779452	779524
602	779957	780029	780101	780173	780245
603	780977	780749	780821	780893	780965

A Table of **Logarithms**,

Num	0	1	2	3	4	Diff
604	781037	781109	781181	781253	781324	72
605	781755	781827	781899	781971	782042	72
606	782473	782544	782616	782688	782759	72
607	783189	783260	783332	783403	783475	71
608	783904	783975	784046	784118	784189	71
609	784617	784689	784760	784831	784902	71
610	785330	785401	785472	785543	785615	71
611	786041	786112	786183	786254	786325	71
612	786751	786822	786893	786964	787035	71
613	787460	787531	787602	787673	787744	71
614	788168	788239	788310	788381	788451	71
615	788875	788946	789016	789087	789157	71
616	789581	789651	789722	789792	789863	70
617	790285	790356	790426	790496	790567	70
618	790988	791059	791129	791199	791269	70
619	791691	791761	791831	791901	791971	70
620	792392	792462	792532	792602	792672	70
621	793092	793162	793231	793301	793371	70
622	793790	793860	793930	794000	794070	70
623	794488	794558	794627	794697	794767	70
624	795185	795254	795324	795393	795463	70
625	795880	795949	796019	796088	796158	69
626	796574	796644	796713	796782	796852	69
627	797268	797337	797406	797475	797545	69
628	797960	798029	798098	798167	708236	69
629	798651	798720	798789	798858	798927	69
630	799341	799409	799478	799547	799616	69
631	800029	800098	800167	800236	800305	69
632	800717	800786	800854	800923	800992	69
633	801404	801472	801541	801610	801678	69
634	802089	802158	802226	802295	802363	68
635	802774	802842	802910	802979	803047	68
636	803457	803525	803594	803662	803730	68
637	804139	804208	804276	804344	804412	68
638	804821	804889	804957	805025	805093	68
639	805501	805569	805637	805705	805773	68

Num	5	6	7	8	9	Di
604	781396	781468	781540	781612	781684	7
605	782114	782186	782258	782329	782401	7
606	782831	782902	782974	783046	783117	7
607	783546	783618	783689	783761	783832	71
608	784261	784332	784403	784475	784546	71
609	784974	785045	785116	785187	785259	71
610	785686	785757	785828	785899	785970	71
611	786396	786467	786538	786609	786680	71
612	787106	787177	787248	787319	787390	71
613	787815	787885	787956	788027	788098	71
614	788522	788593	788663	788734	788804	71
615	789228	789299	789369	789440	789510	71
616	789933	790004	790074	790144	790215	7
617	790637	790707	790778	790848	790918	7
618	791340	791410	791480	791550	791620	7
619	792041	792111	792181	792252	792322	7
620	792742	792812	792882	792952	793022	7
621	793441	793511	793581	793651	793721	7
622	794139	794209	794279	794349	794418	7
623	794836	794906	794976	795045	795115	7
624	795532	795602	795672	795741	795810	7
625	796227	796297	796366	796436	796505	6
626	796921	796990	797060	797129	797198	6
627	797614	797683	797752	797821	797890	6
628	798305	798374	798443	798513	798582	6
629	798996	799065	799134	799203	799272	6
630	799685	799754	799823	799892	799961	6
631	800373	800442	800511	800580	800648	6
632	801061	801129	801198	801266	801335	6
633	801747	801815	801884	801952	802021	6
634	802432	802500	802568	802637	802705	6
635	803116	803184	803252	803321	803389	6
636	803798	803867	803935	804003	804071	6
637	804480	804548	804616	804685	804753	6
638	805161	805229	895297	805365	805433	6
639	805840	805908	805976	806044	806112	6

A Table of **Logarithms,**

n.	0	1	2	3	4	Diff
40	806180	806248	806316	806384	806451	68
41	806858	806926	806993	807061	807129	68
42	807535	807603	807670	807738	807805	68
43	808211	808279	808346	808414	808481	67
644	808886	808953	809021	809088	809156	67
645	809560	809627	809694	809762	809829	67
646	810233	810300	810367	810434	810501	67
647	810904	810971	811039	811106	811173	67
648	811575	811642	811709	811776	811843	67
649	812245	812312	812379	812445	812512	67
650	812913	812980	813047	813114	813181	66
651	813581	813648	813714	813781	813848	66
652	814248	814314	814381	814447	814514	66
653	814913	814980	815046	815113	815179	66
654	815578	815644	815711	815777	815843	66
655	816241	816308	816374	816440	816506	65
656	816904	816970	817036	817102	817169	66
657	817565	817631	817698	817764	817830	66
658	818226	818292	818358	818424	818499	66
659	818885	818951	819017	819083	819149	66
660	819544	819610	819676	819741	819807	66
661	820202	820267	820333	820399	820464	66
662	820858	820924	820989	821055	821120	66
663	821514	821579	821645	821710	821776	65
664	822168	822233	822299	822364	822430	65
665	822822	822887	822952	823018	823083	65
666	823474	823539	823605	823670	823735	65
667	824126	824191	824256	824321	824386	65
668	824777	824842	824907	824972	825036	65
669	825426	825491	825556	825621	825686	65
670	826075	826140	826204	826269	826334	65
671	826723	826787	826852	826917	826981	65
672	827369	827434	827499	827563	827628	65
673	828015	828080	828144	828209	828273	64
674	828660	828724	828789	828853	828918	64
675	829304	829368	829432	829497	829561	64

Num	5	6	7	8	9	Di
640	806519	806587	806655	806722	806790	68
641	807197	807264	807332	807400	807467	68
642	807873	807941	808008	808076	808143	68
643	808548	808616	808684	808751	808818	67
644	809223	809290	809358	809425	809492	67
645	809896	809964	810031	810098	810165	67
646	810569	810636	810703	810770	810837	67
647	811240	811307	811374	811441	811508	67
648	811910	811977	812044	812111	812178	67
649	812579	812646	812713	812780	812847	67
650	813247	813314	813381	813448	813514	66
651	813914	813981	814048	814114	814181	66
652	814581	814647	814714	814780	814847	66
653	815246	815312	815379	815445	815511	66
654	815910	815976	816042	816109	816175	66
655	816573	816639	816705	816771	816838	66
656	817235	817301	817367	817433	817499	66
657	817896	817962	818028	818094	818160	66
658	818556	818622	818688	818754	818819	66
659	819215	819281	819347	819412	819478	66
660	819873	819939	820004	820070	820136	66
661	820530	820596	820661	820727	820792	66
662	821186	821251	821317	821382	821448	66
663	821841	821906	821972	822037	822103	65
664	822495	822560	822626	822691	822756	65
665	823148	823213	823279	823344	823409	65
666	823800	823865	823930	823996	824061	65
667	824451	824516	824581	824646	824771	65
668	825101	825166	825231	825296	825361	65
669	825751	825815	825880	825945	826010	65
670	826399	826464	826528	826593	826658	65
671	827046	827111	827175	827240	827305	65
672	827692	827757	827821	827886	827951	65
673	828338	828402	828466	828531	828596	64
674	828982	829046	829111	829175	829240	64
675	829625	829690	829754	829818	829882	64

A Table of Logarithms,

Num	0	1	2	3	4	Diff
676	829947	830011	830075	830139	830204	64
677	830589	830653	830717	830781	830845	64
678	831230	831294	831358	831422	831486	64
679	831870	831934	831998	832062	832126	64
680	832509	832573	832637	832700	832764	64
681	833147	833211	833275	833338	833402	64
682	833784	833848	833912	833975	834039	64
683	834421	834484	834548	834611	834675	64
684	835056	835120	835183	835247	835310	63
685	835691	835754	835817	835881	835944	63
686	836324	836387	836451	836514	836577	63
687	836957	837020	837083	837146	837209	63
688	837588	837652	837715	837778	837841	63
689	838219	838282	838345	838408	838471	63
690	838849	838912	838975	839038	839101	63
991	839478	839541	839604	839667	839729	63
692	840106	840169	840232	840294	840357	63
693	840733	840796	840859	840921	840984	63
694	841359	841422	841485	841547	841610	63
695	841985	842047	842110	842172	842235	62
696	842609	842672	842734	842796	842859	62
697	843233	843295	843357	843420	843482	62
698	843855	843918	843980	844042	844104	62
699	844477	844539	844601	844664	844726	62
700	845098	845160	845222	845284	845346	62
701	845718	845780	845842	845904	845966	62
702	846337	846399	846461	846523	846585	62
703	846955	847017	847079	847141	847202	62
704	847573	847634	847696	847758	847819	62
705	848189	848251	848312	848374	848435	62
706	848805	848866	848928	848989	849051	61
707	849419	849481	849542	849604	849665	61
708	850033	850095	850156	850217	850279	61
709	850646	850707	850769	850830	850891	61
710	851258	851319	851381	851442	851503	61
711	851869	851931	851992	852053	852114	61

Num	5	6	7	8	9	Dif
676	830268	830332	830396	830460	830525	64
677	830909	830973	831037	831102	831166	64
678	831550	831614	831678	831742	831806	64
679	832189	832253	832317	832381	832445	64
680	832828	832892	832956	833019	833083	64
681	833466	833530	833593	833657	833721	64
682	834103	834166	834230	834294	834357	64
683	834738	834802	834866	834929	834993	64
684	835373	835437	835500	835564	835627	63
685	836007	836071	836134	836197	836261	63
686	836641	836704	836767	836830	836894	63
687	837273	837336	837399	837462	837525	63
688	837904	837969	838030	838093	838156	63
689	838534	838597	838660	838723	838786	63
690	839164	839237	839289	839352	839415	63
691	839792	839855	839918	839981	840043	63
692	840420	840482	840545	840608	840671	63
693	841046	841109	841172	841234	841297	63
694	841672	841735	841797	841860	841922	63
695	842297	842360	842422	842484	842547	62
696	842921	842983	843046	843108	843170	62
697	843544	843606	843669	843731	843793	62
698	844166	844229	844291	844353	844415	62
699	844788	844850	644912	844974	845036	62
700	845408	845470	845532	845594	845656	62
701	846028	846090	846151	846213	846275	62
702	846646	846708	846770	846832	846894	62
703	847264	847326	847388	847449	847511	62
704	847881	847943	848004	848066	848127	62
705	848497	848559	848620	848682	848743	62
706	849112	849174	849235	849297	849358	61
707	849726	849788	849849	849911	849972	61
708	850340	850401	850462	850524	850585	61
709	850952	851014	851075	851136	851197	61
710	851564	851625	851686	851747	851809	61
711	852175	852236	852297	852358	852419	61

Num	0	1	2	3	4	*Diff*
712	852480	852541	852602	852663	852724	61
713	853089	853150	853211	853272	853333	61
714	843698	853759	853820	853881	853941	61
715	854306	854367	854428	854488	854549	61
716	854913	854974	855034	855095	855156	61
717	855519	855580	855640	855701	855761	61
718	856124	856185	856245	856306	856366	60
719	856729	856789	856850	856910	856970	60
720	857333	857393	857453	857513	857574	60
721	857935	857995	858056	858116	858176	60
722	858537	858597	858657	858718	858778	60
723	859138	859198	859258	859318	859378	60
724	859739	859799	859859	859918	859978	60
725	860338	860398	860458	860518	860578	60
726	860937	860996	861056	861116	861176	60
727	861534	861594	861654	861714	861773	60
728	862131	862191	862251	862310	862370	60
729	862727	862787	862847	862906	862966	60
730	863323	863382	863442	863501	863561	59
731	863917	863977	864036	864096	864155	59
732	864511	864570	864630	864689	864748	59
733	865104	865163	865223	865282	865341	59
734	865696	865755	865814	865873	865933	59
735	866287	866346	866405	866465	866524	59
736	866878	866937	866996	867055	867114	59
737	867467	867526	867585	867644	867703	59
738	868056	868115	868174	898233	868292	59
739	868644	868703	868762	868821	868879	59
740	869232	869290	869349	869408	869466	59
741	869818	869877	869935	869994	870053	59
742	870404	870462	870521	870579	870638	58
743	870989	871047	871106	871164	871223	58
744	871573	871631	871690	871748	871806	58
745	872156	872215	872273	872331	872389	58
746	872739	872797	872855	872513	872972	58
747	873321	873379	873437	873495	873553	58

Num	5	6	7	8	9	Di
712	852785	852846	852907	852968	853029	61
713	853394	853455	853516	853576	853637	61
714	854002	854063	854124	854184	854245	61
715	854610	854670	854731	854792	854852	61
716	855216	855277	855337	855398	855459	61
717	855822	855882	855943	856003	856064	61
718	856427	856487	856548	856608	856668	60
719	857031	857091	857151	857212	857272	60
720	857634	857694	857754	857815	857875	60
721	858236	858296	858357	858417	858477	60
722	858838	858898	858958	859018	859078	60
723	859439	859499	859559	859619	859679	60
724	860038	860098	860158	860218	860278	60
725	860637	860697	860757	860817	860877	60
726	461236	861295	861355	861415	861475	60
727	861833	861893	861952	862012	862072	60
728	862430	862489	862549	862608	862668	60
729	863025	863085	863144	863204	863263	60
730	863620	863680	863739	863798	863858	59
731	864214	864274	864333	864392	864452	59
732	864808	864867	864926	864985	865045	59
733	865400	865459	865518	865578	865637	59
734	865992	866051	866110	866169	866228	59
735	866583	866642	866701	866760	866819	59
736	867173	867232	867291	867350	867409	59
737	867762	867821	867880	867939	867997	59
738	868350	868409	868468	868527	868586	59
739	868938	868997	869056	869114	869173	59
740	869525	869584	869642	869701	869760	59
741	870111	870170	870228	870287	870345	59
742	870696	870755	870813	870872	870930	58
743	871281	871339	871398	871456	871515	58
744	871865	871923	871981	872040	872098	58
745	872448	872506	872564	872622	872681	58
746	873030	873088	873146	873204	873262	58
747	873611	873669	873727	873785	873843	58

A Table of Logarithms,

Num	0	1	2	3	4	Diff
748	873902	873969	874018	874076	874134	58
749	874482	874540	874598	874656	874714	58
750	875061	875119	875177	875235	875293	58
751	875640	875698	875756	875813	875871	58
752	876218	876276	876333	876391	876449	58
753	876795	876853	876910	876968	877026	58
754	877371	877429	877487	877544	877602	58
755	877947	878004	878062	878119	878177	57
756	878522	878579	878637	878694	878751	57
757	879096	879153	879211	879268	879325	57
758	879669	879726	879784	879841	879898	57
759	880242	880299	880356	880413	880471	57
760	880814	880871	880928	880985	881042	57
761	881385	881442	881499	881556	881613	57
762	881955	882012	882069	882126	882183	57
763	882524	882581	882638	882695	882752	57
764	883093	883150	883207	883264	883321	57
765	883661	883718	883775	883832	883888	57
766	884229	884285	884342	884399	884455	57
767	884795	884852	884909	884965	885022	57
768	885361	885418	885474	885531	885587	57
769	885926	885983	886039	886096	886152	56
770	886491	886547	886604	886660	886716	56
771	887054	887111	887167	887223	887280	56
772	887617	887674	887730	887786	887842	56
773	888179	888236	888292	888348	888404	56
774	888741	888797	888853	888909	888965	56
775	889302	889358	889414	889470	889526	56
776	889862	889918	889974	890030	890086	56
777	890421	890477	890533	890589	890644	56
778	890980	891035	891091	891147	891203	56
779	891537	891593	891649	891705	891760	56
780	892095	892150	892206	892262	892317	56
781	892651	892707	892762	892818	892873	56
782	893207	893262	893318	893373	893429	56
783	893762	893817	893873	893928	893984	55

Num	5	6	7	8	9	*Diff*
748	874192	874250	874308	874365	874424	58
749	874772	874830	874887	874945	875003	58
750	875351	875409	875466	875524	875582	58
751	875929	875987	876044	876102	876160	58
752	876506	876564	876622	876680	876737	58
753	877083	877141	877198	877256	877314	58
754	877659	877717	877774	877832	877889	58
755	878234	878292	878349	878407	878464	57
756	878809	878866	878924	878981	479038	57
757	879383	879440	879497	879555	879612	57
758	879956	880013	880070	880127	880185	57
759	880528	880585	880642	880699	880756	57
760	881099	881156	881213	881270	881328	57
761	881670	881727	881784	881841	881898	57
762	882240	882297	882354	882411	883468	57
763	882809	882866	882923	882580	883036	57
764	883377	883434	883491	883548	883605	57
765	883945	884002	884059	884115	884172	57
766	884512	884569	884625	884682	884739	57
767	885078	885135	885191	885248	885305	57
768	885644	885700	885757	885813	885870	57
769	886209	886265	886321	886378	886434	56
770	886773	886829	886885	886942	886998	56
771	887336	887392	887449	887505	887561	56
772	887898	887955	888011	888067	888123	56
773	888460	888516	888573	888629	888685	56
774	889021	889077	889133	889190	889246	56
775	889582	889638	889694	889750	889806	56
776	890141	890197	890253	890309	890365	56
777	890700	890756	890812	890868	890924	56
778	891259	891314	891370	891426	891482	56
779	891816	891872	891928	891983	892039	56
780	892373	892428	892484	892540	892595	56
781	892929	892985	893040	893096	893151	56
782	893484	893540	893595	893651	893706	56
783	894039	894094	894150	894205	894261	55

A Table of Logarithms,

Num	0	1	2	3	4	Dif
784	894316	894371	894427	894482	894538	55
785	894870	894925	894980	895036	895091	55
786	895422	895478	895533	895588	895643	55
787	895975	896030	896085	896140	896195	55
788	896526	896581	896636	896691	896747	55
789	897077	897132	897187	897242	897297	55
790	897627	897682	897737	897792	897847	55
791	898176	898231	898286	898341	898396	55
792	898725	898780	898835	898890	898944	55
793	899273	899328	899383	899437	899492	55
794	899820	899875	899930	899985	900039	55
795	900367	900422	900476	900531	900586	55
796	900913	900968	901022	901077	901131	55
797	901458	901513	901567	901622	901676	54
798	902003	902057	902112	902165	902220	54
799	902547	902601	902655	902710	902764	54
800	903090	903144	903198	903253	903307	54
801	903632	903687	903741	903795	903849	54
802	904174	904228	904283	904337	904391	54
803	904715	904770	904824	904878	904932	54
804	905276	905310	905364	905418	905472	54
805	905796	905850	905904	905958	906012	54
806	906335	906389	906443	906497	906550	54
807	906873	906927	906981	907035	907089	54
808	907411	907465	907519	907573	907626	54
809	907948	908002	908056	908109	908163	54
810	908485	908539	908592	908646	908699	54
811	909021	909074	909128	909181	909235	54
812	909556	909609	909663	909716	909770	53
813	910090	910144	910197	910251	910304	53
814	910624	910678	910731	910784	910838	53
815	911158	911211	911264	911317	911371	53
816	911690	911743	911797	911850	911903	53
817	912222	912275	912323	912381	912435	53
818	912753	912806	912859	912913	912966	53
819	913283	913337	913390	913443	913496	53

Num	5	6	7	8	9	Dif
784	894593	894648	894704	894759	894814	5
785	895146	895201	895257	895312	895367	5
786	895699	895754	895809	895864	895919	5
787	896251	896306	896361	896416	896471	5
788	896802	896857	896912	896967	897022	5
789	897352	897407	897452	897517	897572	5
790	897902	897957	898012	898067	898122	5
791	898451	898506	898561	898615	898670	5
792	898999	899054	899109	899164	899218	5
793	899547	899602	899656	899711	899766	5
794	900094	900149	900203	900258	900312	55
795	900640	900695	900749	900804	900859	55
796	901186	901240	901295	901349	901404	55
797	901731	901785	901840	901894	901948	54
798	902275	902329	902384	902438	902492	54
799	902818	902873	902927	902981	903036	54
800	903361	903416	903470	903524	903578	54
801	903903	903958	904012	904066	904120	54
802	904445	904499	904553	904607	904661	54
803	904986	905040	905094	905148	905202	5
804	905526	905580	905634	905688	905742	5
805	906065	906119	906173	906227	906281	5
806	906604	906658	906712	906766	906820	5
807	907142	907196	907250	907304	907358	5
808	907680	907734	907787	907841	907895	5
809	908217	908270	908324	908378	908431	5
810	908753	908807	908800	908914	908967	5
811	909288	909342	909395	909449	909502	5
812	909823	909877	909930	909984	910037	5
813	910358	910411	910464	910518	910571	5
814	910891	910944	910998	911051	911104	5
815	911424	911477	911530	911584	911637	
816	911956	912009	912063	912116	912169	
817	912488	912541	912594	912647	912700	
818	913019	913072	913125	913178	913231	
819	913549	913602	913655	913708	913761	

A Table of Logarithms,

Num	0	1	2	3	4	Diff
820	913814	913867	913920	913973	914026	53
821	914343	914396	914449	914502	914555	53
822	914872	914925	914977	915030	915583	53
823	915400	915453	915505	915558	915611	53
824	915927	915980	916033	916085	916138	53
825	916454	916507	916559	916612	916664	53
826	916980	917033	917085	917138	917190	53
827	917505	917558	917611	917663	917715	53
828	918030	918083	918135	918188	918240	53
829	918554	918607	918659	918712	918764	53
830	919078	919130	919183	919235	919287	52
831	919601	919653	919706	919758	919810	52
832	920123	920175	920228	920280	920332	52
833	920645	920697	920749	920801	920853	52
834	921166	921218	921270	921322	921374	52
835	921686	921738	921790	921842	921894	52
836	922206	922258	922310	922362	922414	52
837	922725	922777	922829	922881	922933	52
838	923244	923296	923348	923399	923451	52
839	923762	923814	923865	923917	923969	52
840	924279	924331	924383	924434	924486	52
841	924796	924848	924899	924951	925002	52
842	925312	925364	925415	925467	925518	52
843	925828	925879	925931	925982	926034	51
844	926342	926394	926445	926497	926548	51
845	926857	926908	926959	927011	927062	51
846	927370	927422	927473	927524	927576	51
847	927883	927935	927986	928037	928088	51
848	928396	928447	928498	928549	928601	51
849	928908	928959	929010	929061	929112	51
50	926419	929470	929521	929572	929623	51
51	929930	929981	930032	930083	930134	51
52	930440	930491	930541	930592	930643	51
53	930949	931000	931051	931102	931153	51
54	931458	931509	931560	931610	931661	51
55	931966	932017	932068	932118	932169	51

Num	5	6	7	8	9	Diff
820	914079	914132	914184	914237	914290	53
821	914608	914660	914713	914766	914819	53
822	915136	915189	915241	915294	915347	53
823	915664	915716	915769	915822	915874	53
824	916191	916243	916296	916349	916401	53
825	916717	916770	916822	916875	916927	53
826	917243	917295	917348	917400	917453	53
827	917768	917820	917873	917925	917978	53
828	918292	918345	918397	918459	918502	53
829	918816	918869	918921	918973	919026	53
830	919340	919392	919444	919496	919549	52
831	919862	919914	919967	920019	920071	52
832	920384	920436	920489	920541	920593	52
833	920906	920958	921010	921062	921114	52
834	921426	921478	921530	921582	921634	52
835	921946	921998	922050	922102	922154	52
836	922466	922518	922570	922622	922674	52
837	922985	923037	923088	923140	923192	52
838	923503	923555	923607	923658	923710	52
839	924021	924072	924124	924176	924228	52
840	924538	924589	924641	424693	924744	52
841	925054	925106	925157	925209	925261	52
842	925570	925621	925673	925725	925776	52
843	926085	926137	926188	926239	926291	51
844	926600	926651	926702	926754	926805	51
845	927114	927165	927216	927268	927319	51
846	927627	927678	927730	927781	927832	51
847	928140	928191	928242	928293	928345	51
848	928652	928703	928754	928805	928856	51
849	929163	929214	929266	929317	929364	51
850	929674	929725	929776	929827	929878	51
851	930185	930236	930287	930338	930389	51
852	930694	930745	930796	930847	930898	51
853	931203	931254	931305	931356	931407	51
854	931712	931763	931814	931864	931915	51
855	932220	932271	932322	932372	932423	51

A Table of Logarithms,

um	0	1	2	3	4	Diff
56	932474	932524	932575	932626	932677	51
57	932981	933031	933082	933133	933183	51
58	933487	933538	933588	933639	933690	51
59	933993	934044	934094	934145	934195	51
60	934498	934549	934599	934650	934700	50
61	935003	935054	935104	935154	935205	50
62	935507	935558	935608	935658	935709	50
63	936011	936061	936111	936162	936212	50
64	036514	936564	936614	936665	936715	50
65	937016	937066	937116	937167	937217	50
66	937518	937568	937618	937668	937718	50
67	938019	938069	938119	938169	938219	50
68	938520	938570	938620	938670	938720	50
69	939020	939070	939120	939170	939220	50
70	939519	939569	939619	939669	939719	50
71	940018	940068	940118	940168	940218	50
72	940516	940566	940616	940666	940716	50
73	741014	941064	941114	941163	941213	50
74	941511	941561	941611	941660	941710	50
75	942008	942058	942107	942157	942206	50
76	942504	942554	942603	942953	942702	50
77	943000	943049	943096	943148	943198	49
78	943494	943544	943593	943643	943692	49
79	943989	944038	944088	944137	944186	49
80	944483	944532	944581	944631	944680	49
81	944976	945025	945074	945124	945173	49
82	945469	945518	945567	945616	945665	49
83	945961	946010	945059	946108	946157	49
84	946452	946501	946551	946600	946649	49
85	946943	946992	947041	947090	947139	49
86	947434	947483	947532	947581	947630	49
87	947924	947973	948022	948070	948119	49
88	948413	948462	948511	948560	948609	49
89	948902	948951	948999	949048	909097	49
90	949390	949439	949488	949536	949585	49
91	949878	949926	949975	950024	950073	49

Num	5	6	7	8	9	Diff
856	932727	932778	932829	932879	932930	51
857	933234	933285	933335	933385	933437	51
858	933740	933791	933841	933892	933943	51
859	934246	934296	934347	934397	934448	51
860	934751	934801	934852	934902	934953	50
861	935255	935306	935356	935406	935457	50
862	935759	935809	935860	935910	935960	50
863	936262	936313	936363	936413	936463	50
864	936765	936815	936865	936916	936966	50
865	937267	937317	937367	937418	937468	50
866	937769	937819	937869	937919	937969	50
867	938269	938319	938370	938420	938470	50
868	938770	938820	938870	938920	938970	50
869	939270	939320	939369	939420	939470	50
870	939769	939819	939868	939918	939968	50
871	940267	940317	940367	940417	940467	50
872	940765	940815	940865	540915	940964	50
873	641263	941313	941362	941412	941462	50
874	941760	941809	941859	941909	941958	50
875	942256	942306	942355	942405	942454	50
876	942752	942801	942851	942900	942950	50
877	943247	943297	943346	943396	943445	49
878	943742	943791	943841	943890	943939	49
879	944236	944285	944335	944384	944433	49
880	944729	944779	944828	944877	944927	49
881	945222	945272	945321	945370	945419	49
882	945715	945764	945813	945862	945911	49
883	946207	946256	946305	946354	946403	49
884	946698	946747	946796	946845	946894	49
885	947189	947238	947287	947336	947385	49
886	947679	947728	947777	947826	947875	49
887	948168	948217	948266	948315	948364	49
888	948657	948706	948755	948804	948853	49
889	949146	949195	949244	949292	949341	49
890	949633	949683	949731	949780	949829	49
891	950121	950170	950219	950267	950316	49

Num	0	1	2	3	4	Diff
892	950365	950414	950462	950511	950560	49
893	950851	950900	950949	950997	951046	49
894	951337	951386	951435	951483	951532	49
895	951823	951872	951920	951969	952017	49
896	952308	952356	952405	952453	952502	48
897	952792	952841	952889	952938	952986	48
898	953276	953325	953373	953421	953470	48
899	953760	953808	953856	953905	953953	48
900	954242	954291	954339	954387	954435	48
901	954725	954773	954821	954869	954918	48
902	955207	955255	955303	955351	955399	48
903	955688	955736	955784	955832	955880	48
904	956168	956216	956264	956313	956361	48
905	956649	956697	956745	956793	956840	48
906	957128	957176	957224	957272	957320	48
907	957607	957655	957703	957751	957799	48
908	958086	958134	958181	958229	958277	48
909	958564	958612	958659	958707	958755	48
910	959041	959089	959137	959184	959232	48
911	959518	959566	959614	959661	959709	48
912	959995	960042	960090	960138	960185	48
913	960471	960518	960566	960613	960661	48
914	960946	960994	961041	961089	961136	47
915	961421	961468	961516	961563	961611	47
916	961895	961943	961990	962038	962085	47
917	962369	962417	962464	962511	962559	47
918	962843	962890	962937	962985	963032	47
919	963315	963363	963410	963457	962504	47
920	963788	963835	963882	963929	963977	47
921	964260	964307	964354	964401	964448	47
922	964731	964778	964825	964872	964919	47
923	965202	965249	965296	965343	965390	47
924	965672	965719	965766	965813	965860	47
925	966142	966189	966236	966283	966329	47
926	966611	966658	966705	966752	966798	47
927	967080	967127	967173	967220	967267	47

Num	5	6	7	8	9	*Diff*
892	950608	950657	950706	950754	950803	49
893	951095	951143	951192	951240	951289	49
894	951580	951629	951677	951726	951775	49
895	952066	952114	952163	952211	952259	49
896	952550	952599	952647	952696	952744	48
897	953034	953083	953131	953180	953228	48
898	953518	953566	953615	953663	953711	58
899	954001	954049	954098	954146	954194	48
900	954484	954532	954580	954628	954977	48
901	954966	955014	955062	955110	955158	48
902	955447	955495	955543	955591	955640	48
903	955928	955976	956024	956072	956120	48
904	956409	956457	956505	956553	956601	48
905	956888	956936	956984	957032	957080	48
906	957368	957416	957464	957512	957559	48
907	957847	957894	957942	957990	958038	48
908	958325	958373	958421	958468	958516	48
909	958803	958850	958898	958946	958994	48
910	959280	959328	959375	959423	959171	48
911	959757	959804	959852	959900	959947	48
912	960233	960280	960328	960376	960423	48
913	960709	960756	960804	960851	960899	48
914	961184	961231	961279	961326	961374	47
915	961658	961706	961753	961801	961848	47
916	962132	962180	962227	962275	962322	47
917	962606	962653	962701	962748	962795	47
918	963079	963126	963174	963221	963268	47
919	963552	963599	963646	963693	963741	47
920	964024	964071	964118	964165	964212	47
921	964495	964542	964590	964637	964684	47
922	964966	965013	965061	965108	965155	47
923	965437	965484	965531	965578	965625	47
924	965907	965954	966001	966048	966095	47
925	966376	966423	966470	966517	966564	47
926	966845	966892	976939	966986	967033	47
927	967314	967361	967408	967454	967501	47

A Table of Logarithms,

Num	0	1	2	3	4	Diff
928	967548	967595	967642	967688	967735	47
929	968016	968062	968109	968156	968203	47
930	968483	968530	968576	968623	968670	47
931	968950	968996	969043	969090	969136	47
932	969416	969403	969509	969556	969602	47
933	969882	969928	969975	970021	970068	47
934	970347	970393	970440	970486	970533	46
935	970812	970858	970904	970951	970997	46
936	971276	971322	971369	971415	971461	46
937	971740	971785	971832	971879	971925	46
938	972203	972249	972295	972342	972388	46
939	972666	972712	972758	972804	972851	46
940	973128	973174	973220	973266	973313	46
941	973590	973636	973682	973728	973774	46
942	974051	974097	974143	974189	974235	46
943	974512	974558	974604	974650	974696	46
944	974972	975018	975064	975110	975156	46
945	975432	975478	975524	975570	975616	46
946	975891	975937	975983	976029	976075	46
947	976350	976396	976442	976488	976533	46
948	976808	976854	976900	976946	976992	46
949	977266	977312	977358	977403	977449	46
950	977724	977769	977815	977861	977906	46
951	978181	978226	978272	978317	978363	46
952	978637	978683	978728	978774	978819	45
953	979093	979138	979184	979230	979275	45
954	979548	979594	979639	979685	979730	45
955	980003	980049	980094	980140	980185	45
956	980458	980503	980549	980594	980640	45
957	980912	980957	981003	981048	981093	45
958	981366	981411	981456	981501	981547	45
959	981819	981864	981909	981954	982000	45
960	982271	982316	982362	982407	982452	45
961	982723	982769	982814	982859	982904	45
962	983175	983220	983265	983310	983356	45
963	983626	983671	983716	983762	983807	45

Num	5	6	7	8	9	Dif
928	967782	967829	967875	967922	967969	47
929	968249	968296	968343	968389	968436	47
930	968716	968763	968810	968856	968903	47
931	969183	969229	969276	969323	969369	47
932	969649	969695	969742	969789	969835	47
933	970114	970161	970207	970254	970300	47
934	970579	970626	970672	970719	970765	4
935	971044	971090	971137	971183	971229	4
936	971508	971554	971601	971647	971693	4
937	971971	972018	972064	972110	972157	4
938	972434	972481	972527	972573	972619	4
939	972897	972943	972989	973035	973082	4
940	973359	973405	973451	973497	973543	4
941	973820	973866	973913	973959	974005	4
942	974281	974327	974374	974420	974466	4
943	974742	974788	974834	974880	974926	4
944	975202	975248	975294	975340	975386	4
945	975662	975707	975753	975799	975845	4
946	976121	976167	976212	976258	976304	4
947	976579	976625	976671	976717	976763	4
948	977037	977083	977129	977175	977220	4
949	977495	977541	977586	977632	977678	4
950	977952	977998	978043	978089	978135	4
951	978409	978454	978500	978546	978591	4
952	978865	978911	978956	479002	979047	4
953	979321	979365	979412	979457	979503	4
954	979776	979821	979867	979912	979958	4
955	980231	980276	980322	980367	980412	4
956	980685	980730	980776	980821	980867	4
957	981139	981184	981229	981275	981320	4
958	981592	981637	981683	981728	981773	4
959	982045	982090	982135	982181	982226	4
960	982467	982543	982588	982633	982678	4
961	982949	982994	983040	983085	983130	4
962	983401	983446	983491	983536	983581	4
963	983852	983897	983942	983987	984032	4

A Table of Logarithms,

Num	0	1	2	3	4	Dif
964	984077	984122	984167	984212	984257	45
965	984527	984572	984617	984662	984707	45
966	984977	985022	985067	985112	985157	45
967	985426	985471	985516	985561	985606	45
968	985875	985920	985965	986010	986055	45
969	986324	986369	986413	986458	986503	45
970	986772	986816	986861	986906	986951	45
971	987219	987264	987309	987353	987398	45
972	987666	987711	987756	987800	987845	45
973	988113	988157	988202	988247	988291	45
974	988559	988604	988648	988693	988737	45
975	989005	989049	989094	989138	989183	45
976	989450	989494	989539	989583	989628	44
977	989895	989939	989983	990028	990072	44
978	990339	990383	990428	990472	990516	44
979	990783	990827	990871	990916	990960	44
980	991226	991270	991315	991359	991403	44
981	991669	991713	991757	991802	991846	44
982	992112	992155	992200	992244	992288	44
983	992554	992598	992642	992686	992730	44
984	992995	993039	993083	993127	993172	44
985	993436	993480	993524	993568	993613	44
986	993877	993921	993965	994009	994053	44
987	994317	994361	994405	994449	994493	44
988	994757	994801	994845	994889	994933	44
989	995196	995240	995284	995328	995372	44
990	995635	995679	995723	995767	995811	44
991	996074	996117	996161	996205	996249	44
992	996512	996555	996599	996643	996687	44
993	996949	996993	997037	997080	997124	44
994	997386	997430	997474	997517	997561	44
995	997823	997867	997910	997954	997998	44
996	998259	998303	998346	998390	998434	44
997	998695	998739	998782	998826	998869	44
998	999130	999174	999218	999261	999305	44
999	999565	999609	999652	999696	999739	44

from 1, To 10000.

Num	5	6	7	8	9	Diff
964	984302	984347	984392	984437	984482	45
965	984752	984797	984842	984887	984932	45
966	985202	985247	985292	985337	985382	45
967	985651	985696	985741	985786	985830	45
968	986100	986144	986189	986324	986279	45
969	985548	986593	986637	986682	986727	45
970	986996	987040	987085	987130	987175	45
971	987443	987488	987532	987577	987622	45
972	987890	987934	987979	988024	988068	45
973	988336	988381	988425	988470	988514	45
974	988782	988826	988871	988916	988960	45
975	989227	989272	989316	989361	989405	45
976	989972	989717	989761	989806	989850	44
977	990117	990161	990206	990250	990294	44
978	990561	990605	990650	990694	990738	44
979	991004	991049	991093	991137	991182	44
980	991448	991492	991536	991580	991625	44
981	991890	991934	991979	992023	992067	44
982	992333	992377	992421	992465	992509	44
983	992774	992819	992863	992907	992951	44
984	993216	993260	993304	993348	993392	44
985	993657	993701	993745	993789	993833	44
986	993097	994141	994185	994229	994273	44
987	994537	994581	994625	994669	994713	44
988	994977	995021	995055	995108	995152	44
989	995416	995460	995504	995547	995591	44
990	995854	995898	995942	995986	996030	44
991	996293	996337	996380	996424	996468	44
992	996731	996774	996818	996862	996906	44
993	997168	997212	997255	997299	997343	44
994	997695	997648	997692	997736	997779	44
995	998041	998085	998129	998172	998216	44
996	998477	998521	998564	998608	998652	44
997	998913	998956	999000	999043	999087	44
998	999438	999392	999435	999479	999522	44
999	999783	999826	999870	999913	999956	44